Discussion of:

Interpreting factor models

by:

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Paper Outline

- Success of a factor/characteristic model tells you nothing about whether the underlying economy is rational/behavioral.
- Model showing that sentiment $\Rightarrow$ sentiment factor.
- Asset return premia are described well by a factor model based on the first few principal components from a PCA.
- Factor structure & premia are unstable.

Discussion Outline

- This is a good and interesting analysis.
- It is an important contribution:
  - The economic meaning of the rejection of a factor/characteristics model is often misinterpreted in the finance literature.
- I will have a few quibbles with some of the analysis, but agree with most everything in this paper.

Discussion Outline

- I'm going to talk about the following issues:
  - Factor vs. characteristic models.
  - Review of Daniel and Titman (1997, 2012) analysis.
  - PCA analysis
  - What can we say about behavioral vs. rational models ?

Factors Versus Characteristics

- Given the absence of arbitrage (or LOP):
  1. There exists a factor model that prices all assets perfectly.
  2. There exists a characteristics model that prices all assets perfectly.
- Thus, the rejection of a particular factor model (*e.g.*, the FF(1993) model) doesn't prove that prices are set by rational or irrational agents.
  - It just demonstrates that the mean variance efficient portfolio isn't spanned by the factors of the particular factor model considered.

Factor Model Existence

- In the absence of arbitrage or, equivalently, assuming the law of one price holds:

$$\mathbb{E}[\tilde{m}\tilde{R}_i] = 0$$

where $\tilde{R}$ is any excess return (*i.e.* on a Long-Short portfolio). Then the LS portfolio which is maximizes the correlation with $-\tilde{m}$ is the highest possible Sharpe-ratio portfolio:

$$\frac{\mathbb{E}[\tilde{R}]}{\sigma_R} = -\rho_{m,R}\,\frac{\sigma_m}{\mathbb{E}[\tilde{m}]}$$

  - This portfolio is therefore MVE, and prices all LS portfolios.
  - If the MVE portfolio is spanned by the factors of the factor model, then the factor model will price all assets.

Characteristics Model Existence

- Similarly, given no-arbitrage (or LOP), and therefore the existence of an MVE portfolio:

$$\mathbb{E}[\tilde{R}_i] = \beta_{i,MVE} \cdot \mathbb{E}[\tilde{R}_{MVE}]$$

- If we define the vector of asset characteristics θ_i appropriately, a linear combination of the characteristics will also perfectly explain the excess returns of all assets.

Characteristics Model Existence

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$$\mathbb{E}[\tilde{R}_i] = \underbrace{\beta_{i,MVE}}_{\propto \mathbf{b}'\boldsymbol{\theta}_i} \mathbb{E}[\tilde{R}_{MVE}]$$

- If we define the vector of asset characteristics $\boldsymbol{\theta}_i$ appropriately, a linear combination of the characteristics will also perfectly explain the excess returns of all assets.

What can factor/characteristic models tell us?

- What can we conclude?
  - Nothing, other than that the LOP holds!
  - To saw more we need a model of preferences/state-prices.
- From Hansen and Jagannathan (1991), $\mathbb{E}[\tilde{m}\tilde{R}_i] = 0 \Rightarrow$:

$$\left(\frac{E[\tilde{R}]}{\sigma_R}\right) = -\rho_{m,R}\left(\frac{\sigma_m}{E[m]}\right)$$

even without a precise model of preferences, we can conclude that:

- A really high Sharpe-ratio implies a really high σ_m
- The MVE portfolio should be highly correlated with proxies for marginal utility.

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- So even without specifying a precise model, it is worthwhile seeing how high a Sharpe-ratio is possible using information that we think investors might not process properly.

Model with Behavioral and Rational Investors

- See Daniel, Hirshleifer, and Subrahmanyam (2001).
- What if we have both behavioral and overconfident/sentiment investors?
- In a CARA-Normal setting with agents with different beliefs, prices will reflect a weighted average of the discounted expected payoff of the assets.
- If the measure of rational agents is ≈ 1:
  - Prices will be almost exactly what they would be were all agents rational
  - Overconfident agents (incorrectly) will expect high Sharpe-ratios.

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Fama and French (1993)

- The Daniel and Titman (1997) “characteristics” model was very much a response to Fama and French (1993).
- FF (1993) tests were interpreted as evidence that the three-factor model (MKT, HML, and SMB) provided a good summary of equity returns.
- This was based on their empirical tests showing that the three factors(MKT, HML, and SMB) priced the (now famous) FF 25 Sz-BM sorted portfolios.

Fama and French (1993)

- We argued that their tests had low statistical power against interesting alternatives.
- To assess power, you need an alternative hypothesis so we propose three return generating processes:
  1. A time-invariant factor model
  2. A factor model with time varying factor loadings
  3. A pure characteristics model (with asymptotic arbitrage)
- We argued that under *any* of these three models you would get the FF(93) empirical results.

Fama and French (1993)

- We further argued that the problem with the FF 93 tests was the low dimensionality of the asset return space.
- if you sort into portfolios on the basis of size and BM, you eliminate a lot of the underlying factor structure.
- For example, if the RGP is the characteristics model, you will come up with three factors (a "level" or market factor, a size factor and a bm factor), even when the set of equities is governed by a far richer factor structure.
  - The R^2s for time-series regressions of the FF-25 portfolios on the 3 factors are mostly $> 90\%$.
    - See Lewellen, Nagel, and Shanken (2010) and Daniel and Titman (2012).

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    - See Lewellen, Nagel, and Shanken (2010) and Daniel and Titman (2012).
- However, if you expand the asset space, you find that you can pretty easily reject the FF (3-factor) model.
  - We note that this *doesn't* mean that you can reject all factor models.
  - It *does* mean that the MVE portfolio has a higher SR than just a combination of Mkt, HML and SMB.

PCA Analysis

- This paper does principal components analysis and shows that a low-order principal components model explains returns well.
- this is the one part of their analysis that I really don't like.
- The problem is that any time you sort on the basis of some characteristic into portfolios you eliminate the factor structure that is not directly associated with that characteristic.
- They do their tests with the FF 25 portfolios or the Novy-Marx and Velikov (2014) portfolios.
- When this is done with the FF 25 portfolios, the results are logically equivalent to the original Fama French findings and are wrong.
    - If the authors are going to do this test they should use a different set of portfolios.

Strategy Sharpe Ratios

Below are the *ex-post* Sharpe Ratios (1963:01-2014:05) tangency portfolios based on:

- The Fama and French (1993) portfolios (Mkt, SMB, HML)
- The Carhart (1997) price momentum portfolio UMD.
- Daniel and Titman (2006) Issuance & Accrual portfolios.
- Two low-volatility factor portfolios:
  - Frazzini and Pedersen (2013) and Ang, *et. al.* (2006).

Portfolio Weights (%)								Sharpe Ratio
Mkt-Rf	SMB	HML	UMD	ISU	ACR	BAB	IVOL	
100.0	–	–	–	–	–	–	–	0.40
35.1	19.7	47.2	–	–	–	–	–	0.78
26.0	10.3	33.2	30.5	–	–	–	–	1.09
8.6	4.5	34.2	17.8	26.3	8.7	–	–	1.38
7.6	12.2	14.2	4.7	17.7	9.9	9.5	23.7	1.78

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Caveats

1. The SRs on the last page are *ex-post* optimal portfolios.
2. IVOL (and BAB) are potentially faster/harder to trade than the other factors
   - other factors are VW; all (ex. UMD) are rebalanced annually
3. These factors weren't know in 1963, and as a result of competition strategy performance will likely decile over time.

Start Date†	Factors	Weighting	SR
1963:01	All	Opt.	1.78
–	All	EW	1.54
–	No Vol	EW	1.05
2000:01	No Vol	EW	0.76
2007:01	No Vol	EW	0.87

† All sample periods end in 2014:05. EW SR's for *all* factors: 1.30 post '00; 1.04 post '07.

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