

# The Dynamics of Disagreement \*

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## Abstract

We infer how the estimates of firm value by “optimists” and “pessimists” evolve in response to information shocks by examining returns and disagreement measures for portfolios of short-sale constrained stocks which have experienced large gains or large losses. Our analysis suggests the presence of two groups, one of which overreacts to new information and remains biased over about five years, and a second group which underreacts and whose expectations are unbiased after about one year. Our results have implications for the belief dynamics that underly the momentum and long-term reversal effect.

Keywords: disagreement, differences of opinion, short-sale constraints, momentum, long-term reversal, mispricing persistence

JEL-Classification: G12, G14

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In the last several decades, financial economists have begun to carefully examine the consequences of disagreement or *differences of opinion*. In part, this attention can be motivated by the large trading volumes in securities markets, which are difficult to explain without large levels of and changes in disagreement (see, e.g., [Hong and Stein, 2007](#)). In this paper, we attempt to measure how the distribution of beliefs about security values evolves over time in response to shocks. That is, we examine the dynamics of disagreement.

When opinions differ regarding the value of a security, and investors can costlessly short-sell that security, the security price will reveal only the weighted mean of the distribution of beliefs, and nothing about the dispersion of agents' beliefs. However, if a security becomes hard to borrow, then the price will reflect only the beliefs of the most optimistic agents ([Miller, 1977](#)). Moreover, as we show here, using the cost of borrowing, we can learn about the dispersion of beliefs.

For example, consider a constant absolute risk aversion (CARA)-normal framework with a single security that will pay an uncertain liquidating dividend  $\tilde{D}$  in one period. For simplicity, assume the risk-free rate is zero and that risk aversion is small (so that we can ignore risk-premia). Most importantly, assume only a fixed number of shares are available for lending, and any shares lent and sold cannot be rehypothecated. If a frictionless market exists for lending these shares, the cost of borrowing will be the cost that matches this (inelastic) supply with demand from pessimists who wish to borrow the shares for the purpose of short selling.

In this setting, without disagreement, the price will equal the expected liquidating dividend ( $P = \mathbb{E}[\tilde{D}]$ ), and the short interest as well as the marginal lending fee will both be zero. However, what if there is disagreement? Consider a setting with two masses of investors A and B of equal measure and with equal risk aversion who disagree about the expected payoff. Specifically, assume  $\mathbb{E}^A[\tilde{D}] = \$30$  and  $\mathbb{E}^B[\tilde{D}] = \$20$ , where  $\mathbb{E}^i[\cdot]$  denotes the expectation of an investor of type  $i$ . If the number of shares available for borrowing is large, the security price will be the average of  $\mathbb{E}^A[\tilde{D}]$  and  $\mathbb{E}^B[\tilde{D}]$ : \$25. Note also that if a “wisdom of

crowds” effect is at work, such that the average across agents’ expectations is “rational” (i.e.,  $\mathbb{E}^R[\tilde{D}] = 0.5 \cdot \mathbb{E}^A[\tilde{D}] + 0.5 \cdot \mathbb{E}^B[\tilde{D}] = \$25$ ), the expectation of the security price-change in the period will be zero, because  $\mathbb{E}^R[\tilde{D}] - P = 0$ .

However, according to Miller’s (1977) intuition, when the number of shares available for borrowing is small, the price will reflect primarily the views of the most optimistic agents. In this example, as the number of shares available for borrowing approaches zero, the security price will approach  $\mathbb{E}^A[\tilde{D}] = \$30$ . Also, the borrow cost will approach the difference in the agents’ valuations ( $\mathbb{E}^A[\tilde{D}] - \mathbb{E}^B[\tilde{D}] = \$10$ ): the pessimistic agents in B would be willing to pay up to \$10 to short a share they view as being overvalued by \$10. Here, the rational expectation of the security price-change will be -\$5, so the optimistic agents in A earn a per-share expected payoff of -\$5. However, the pessimistic agents (in B) who short the security also earn an expected payoff per share of -\$5; they benefit from the \$5/share expected price decline but are required to pay the \$10/share borrow cost. Thus, an observer can unambiguously infer the beliefs of both the optimists and pessimists from the price and the borrowing cost of the constrained stock.<sup>1</sup>

We extend this intuition to a multi-period setting and bring it to the data. Specifically, we examine both the returns and proxies for disagreement for constrained securities following information shocks. Doing so allows us to make inferences about how the beliefs of market participants evolve over time in response to these shocks.

Our baseline empirical exercise examines the returns of two portfolios, both of which contain firms that are hard-to-borrow (based on proxies we describe below). Our constrained past winner and past loser portfolios contain firms that experienced, respectively, large positive and negative returns in the year leading up to portfolio formation. In the first year post-formation, we find the portfolios earn market-adjusted returns of -13% and -19%,

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<sup>1</sup>In more complicated settings, the links between optimists’ beliefs and prices as well as pessimists’ beliefs and lending fees are not as direct as in our simple two-type example. Risk-aversion, indirect short selling costs (other than lending fees), and dynamics complicate matters. However, the claim that prices of heavily constrained stocks are set by optimists is fairly general. Similarly, higher borrowing fees will proxy for higher belief dispersion even in more general settings with a continuous distribution of beliefs. In such a setting, the borrow cost will reflect the difference in beliefs between the marginal buyer and the marginal seller.

respectively. This result is consistent with findings in the existing literature (Reed, 2013, provides an excellent survey), which finds the average excess returns of constrained stocks are negative. However, starting one year after portfolio formation, the portfolio of past losers earns a zero alpha, while the portfolio of past winners earns statistically significant negative alphas in years two through five post-formation, and earns a compounded market-adjusted return over these four years of  $-42\%$  ( $t = -7.39$ ).<sup>2</sup> This finding is shown graphically in Panel A of Figure 1 by plotting cumulative abnormal returns (CAR). We confirm this difference in mispricing persistence between constrained winners and losers using a variety of different statistical tests (e.g., Panel B shows bootstrapped confidence intervals for the post-formation CARs) and by using data that is out-of-sample in time (NYSE/AMEX stocks in the 1980s) and out-of-sample in regions (i.e., international firms). Details of the empirical tests are reported in Section 3.

[INSERT Figure 1 HERE]

Interestingly, this asymmetry between constrained past losers and winners in return persistence is not mirrored in disagreement persistence. Panel C in Figure 1 plots the lending fee (from Markit) for our constrained past winner and -loser portfolios over the period from August 2004 through June 2020 for which we have borrow-cost data. Note that borrow costs for the firms in our portfolios are high at portfolio formation and decline subsequently but remain elevated for about five years following both positive and negative shocks.<sup>3</sup>

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<sup>2</sup>The losers' compounded market-adjusted return over this period is  $5\%$  ( $t = 0.44$ ). The compounded market-adjusted return from years two through five post-formation for each portfolio is the difference between the four-year (years two through five post-formation) buy-and-hold-return of the portfolio and the 4-year buy-and-hold-return of the market. The numbers in the text are the averages over all 411 (363 for years two through five) compounded market-adjusted returns for constrained winner and loser portfolios formed during our sample period, respectively. Although the methodology used to calculate the compounded market-adjusted return is different from the cumulative abnormal returns used to compile Figure 1, both methods deliver consistent results.

<sup>3</sup>Constrained losers do not earn negative risk-adjusted returns one year after portfolio formation, although disagreement is still high. This result is at odds with the wisdom-of-crowds hypothesis that the consensus forecast is rational. Specifically, based on the reasoning above, it suggests short sellers of these securities are losing money by paying high shorting fees and are overly pessimistic, whereas the optimistic agents holding these hard-to-borrow securities have approximately correct beliefs. Most empirical results in the short selling literature, including our results for constrained winners, emphasize that short sellers tend to be right and buyers are too optimistic. However, for this portfolio of constrained losers, the consensus view is too pessimistic instead of being too optimistic.

This pattern of long disagreement persistence implied by the lending fees shows up in a similar way for unconstrained winners and losers if we look at disagreement proxies other than lending fees. Panel D shows the average dispersion of analysts' earnings forecasts for winners and losers that are additionally in the top quintile of changes in forecast dispersion over the formation period. We see an increase in analyst disagreement pre-formation and a gradual decrease post-formation, which lasts about five years.

[INSERT Figure 2 HERE]

In summary, we establish three empirical facts about short-sale constrained firms: (1) strong negative abnormal returns following positive price shocks that persist for about five years, (2) strong negative abnormal returns following negative price shocks that persist for about one year, and (3) strong disagreement following both positive and negative price shocks that persists for about five years. What do these empirical findings imply for the impulse responses of optimists' and pessimists' beliefs about firm values? We describe the implications and illustrate them with the use of the stylized plots of impulse responses in Figure 2, which are generated from the model we develop in Section 4. First, the strong negative abnormal returns following a positive price shock suggest the beliefs of the most optimistic agents are too optimistic at time 0 and must decay toward the rational expectation over a roughly five-year period (the upper green line in Figure 2). These optimistic agents appear to overreact to positive information that is part of the time-0 shock.

Second, the shorter-lived negative abnormal returns following the negative price shocks imply that, following these shocks, the beliefs of the most optimistic agents (the lower pink line in Figure 2) are also too optimistic, suggesting these agents initially underreact to the new negative information, but that this underreaction is resolved after only one year.

Finally, the empirical finding that disagreement persists for about five years suggests the distance between the beliefs of the optimists and pessimists decays toward zero over a period of about five years. In Panel B, the blue line plots the level of disagreement coming out of our model, which is roughly consistent with what we see in Panels C and D of Figure 1.

Combined with the optimists' beliefs inferred from prices that we plotted in [Figure 2](#) Panel A, the level of disagreement allows us to identify the pessimists' beliefs, which are just the optimists' beliefs minus the level of disagreement at that time.

[Figure 2](#) Panel B shows graphically that the implied impulse responses of the agents' beliefs to the positive and negative shocks are symmetric, in the sense that the optimist's/pessimist's beliefs following a positive shock are the mirror image of the pessimist's/optimist's beliefs following a negative shock. This assumption is one of the simplest ways to capture the long-lasting disagreement among constrained and unconstrained stocks shown in [Figure 1](#).

What are the implications for the impulse response in prices that would be observed for unconstrained stocks in response to positive and negative price shocks? Again, relying loosely on the intuition from the simple two-type model above, the price would be the average of the beliefs, which would be the dashed blue line in Panel B of [Figure 2](#). These price paths are consistent with the large literature on momentum and subsequent long-term reversals, as discussed extensively in the behavioral literature.

One type of shock that we have not yet discussed is one that is *not* accompanied by an increase in disagreement. In practice, such unambiguous news shocks certainly exist, and firms that have experienced these shocks will enter both winner and loser portfolios. However, for our baseline analysis of short-sale constrained stocks, the fact that we are considering only high short interest securities should filter out most low-disagreement shocks. To the extent that our mechanism partly drives the momentum and long-term reversal effects for unconstrained stocks, these effects should be more muted for low-disagreement shocks. Consistent with this hypothesis, there is compelling evidence that momentum is stronger in high-disagreement ([Zhang, 2006](#)), and low-quality or difficult-to-value information environments ([Daniel and Titman, 1999](#); [Hong, Lim, and Stein, 2000](#); [Cohen and Lou, 2012](#)).

In summary, this paper looks at constrained stocks to learn more about the dynamics of disagreement after large information shocks. High-energy physics provides a useful analogy to our exercise here: physics researchers examine the behavior of fundamental particles at

high energies to test alternative models of particle interactions. By examining matter in extreme settings, they can pull apart the underlying components of these particles. This approach provides them with the additional data they need to develop models that get closer to capturing the underlying mechanisms driving basic physics. Here, we examine “extreme” (constrained) securities in order to infer the distribution of beliefs that would otherwise be aggregated into the price. Results for the set of constrained stocks allow us to look at our existing models for unconstrained stocks from a new and different perspective that challenges existing models of momentum and long-term reversal, as these models are unable to explain the patterns for constrained and unconstrained stocks simultaneously. In [Section 4](#), we briefly relate the empirical results to existing models and propose a new dynamic heterogeneous-agent model featuring overconfidence ([Daniel, Hirshleifer, and Subrahmanyam, 1998](#)) and slow information diffusion ([Hong and Stein, 1999](#)) which is able to both explain this asymmetry in mispricing persistence among short-sale constrained stocks, and to match momentum and long-term reversal effects for unconstrained stocks.

## 1 Related Literature

[Miller \(1977\)](#) argues that when investors disagree about the expected return on a security, and when short selling of that security is restricted, the security will be overpriced. Subsequent empirical research has explored this argument in detail. Consistent with the divergence-of-opinion part of Miller’s argument, firms for which the dispersion of analysts’ forecasts is high tend to earn lower future stock returns ([Danielsen and Sorescu, 2001](#); [Diether, Malloy, and Scherbina, 2002](#)). Moreover, the returns are more negative when disagreement is large and short-sale constraints are binding ([Boehme, Danielsen, and Sorescu, 2006](#)). These negative returns are concentrated around earnings announcements, consistent with the idea that earnings announcements at least partly resolve disagreement ([Berkman, Dimitrov, Jain, Koch, and Tice, 2009](#)). Shocks in the lending market have predictive power for future returns

(Asquith, Pathak, and Ritter, 2005; Cohen, Diether, and Malloy, 2007). Finally, some evidence suggests long-short anomaly strategy profitability tends to be concentrated in the short leg, and in stocks that are expensive to short (Hirshleifer, Teoh, and Yu, 2011; Stambaugh, Yu, and Yuan, 2012; Drechsler and Drechsler, 2016).<sup>4</sup> In a related vein, Engelberg, Reed, and Ringgenberg (2018) relate loan-fee uncertainty and recall risk to price inefficiencies.

D’Avolio (2002) and Geczy, Musto, and Reed (2002) are early papers that examine the stock-lending market using proprietary borrow-cost data. A major takeaway of these studies is that all but a small percentage of common stocks can be borrowed at low cost for short selling purposes. However, Kolasinski, Reed, and Ringgenberg (2013) argue the lending supply curve is almost perfectly elastic at low levels of demand, but above a certain level, it becomes inelastic. They argue that this structure results from a setting where a number of shares is available at almost zero cost. However, once this supply is exhausted, finding additional shares quickly becomes costly. As a result, once demand for borrowing shares increases above a certain threshold, borrowing rates start to increase dramatically.

Because data on borrowing costs are not always available and may not reflect the full set of costs associated with borrowing shares, researchers have used different proxies to identify constrained stocks. Many early papers (Figlewski, 1981; Asquith and Meulbroek, 1996; Dechow, Hutton, Meulbroek, and Sloan, 2001; Desai, Ramesh, Thiagarajan, and Balachandran, 2002) use short interest as a proxy for borrow costs, but this approach has been criticized by Chen, Hong, and Stein (2002). For example, shares of firms with low short interest may or may not be hard-to-borrow depending on the numbers of shares that are available for lending at almost zero cost.

A second set of papers proxies for short-sale constraints with measures of institutional holdings. Chen, Hong, and Stein (2002) use mutual fund holdings, and Nagel (2005) uses residual institutional ownership, the residuals of regressing (a logit-transformation of) institutional ownership onto log-size and log-size squared. The idea is that firms with low

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<sup>4</sup>By contrast, Israel and Moskowitz (2013) provide evidence that momentum, value, and size are robust on the long side and thus do not overly rely on short selling.

institutional holdings are more likely to be constrained. However, similar caveats apply here. Many stocks with close-to-zero short interest may end up getting classified as constrained, even though their short interest is far below the level of institutional ownership, and shorting these stocks is easily possible.

A third set of papers relies directly on loan fees and/or loan quantities (see, e.g., [Jones and Lamont, 2002](#); [Cohen, Diether, and Malloy, 2007](#); [Blocher, Reed, and Van Wesep, 2013](#)). Although this approach delivers reliable information about constraints, it is not suitable for studies interested in long-term returns. Proprietary data sets rely on short sample periods and even Markit data, probably the most comprehensive source of lending-market data, does not provide sufficient coverage for our purposes pre-2004.

We therefore follow [Asquith, Pathak, and Ritter \(2005\)](#) and use a combination of low institutional ownership and high short interest as a characteristic of most constrained firms. Our analysis of Markit reported lending fees confirms this finding. For a shorter sub-sample, we can calculate the Markit indicative and average lending fees and find that, for low institutional ownership firms, such fees are about three to five times higher for firms that also have high short interest at the same time, relative to firms with low short interest (see Panel N of [Table C.7](#) in the Appendix). By contrast, using residual institutional ownership leads to portfolios of firms for which the fee is substantially smaller, on average, even when focusing on high-short-interest firms (see [Table C.8](#), Panel N, in the Appendix). The combined use of low institutional ownership and high short interest is also consistent with our model presented in [Section 4.2](#), the model by [Blocher, Reed, and Van Wesep \(2013\)](#), and the empirical results reported in [Asquith, Pathak, and Ritter \(2005\)](#) and [Cohen, Diether, and Malloy \(2007\)](#).

To our knowledge, no one in the literature has simultaneously analyzed past returns and disagreement proxies, as we do here, with the goal of analyzing the evolution of the dynamic response of disagreement to large information shocks. However, a few studies have reported longer-term returns of short-sale constrained stocks ([Chen, Hong, and Stein, 2002](#); [Nagel,](#)

2005; Lamont, 2012; Weitzner, 2020). The results of these studies are broadly consistent with our hypotheses. Specifically, Nagel (2005) documents short-term overreaction to good cash-flow news and short-term underreaction to bad cash-flow news among stocks with low residual institutional ownership and that prices do not reverse over a three-year period post-formation.

Our empirical approach is based on the premise that large disagreement shocks coincide with large price shocks. The approach has a natural link to theories of momentum (see, e.g., the model in Hong and Stein, 2007). In Section 4, we point out that our empirical results are inconsistent with existing explanations (see Subrahmanyam, 2018, for a survey) and with straightforward extensions of these explanations. We further present the intuition of a new model based on established behavioral concepts that provides a first explanation of the new empirical evidence, and we relate this model to existing theoretical approaches.

Our paper further speaks to the ongoing debate regarding whether bubbles are empirically identifiable. The empirical challenge in identifying asset-pricing bubbles has been the lack of observability of the fundamental value, which leads to the joint hypothesis problem (Fama, 1970). Recent work by Greenwood, Shleifer, and You (2019) shows sharp price increases of industries, along with certain characteristics of this run-up, help forecast the probability of crashes and thereby help identify and time a bubble. Our analysis of constrained winners adds to this strand of literature, as we show, on an individual stock basis, that price run-ups can be used to forecast low future returns when paired with indications of limits to arbitrage. Our theoretical and empirical approach can be interpreted as a methodology for identifying individual stock bubbles, and determining the decay rates of these bubbles.

## 2 Data

We collect returns (monthly and daily), market capitalization, and volume data from the Center for Research in Security Prices (CRSP). Our sample consists of all common ordinary NYSE, AMEX, and NASDAQ stocks from May 1980 through June 2020.<sup>5</sup>

In the next section, we form portfolios at the start of month  $t$  based on a number of firm-specific variables. The first sorting variable is each firm’s cumulative *past return* from month  $t - 12$  through month  $t - 2$ . This measure of momentum is the one used in [Carhart \(1997\)](#), [Fama and French \(2008\)](#), and numerous other studies.

The second sorting variable is the *institutional ownership ratio (IOR)*. IOR is based on Thomson-Reuters Institutional 13-F filings until June 2013, and on WRDS-collected SEC data after June 2013.<sup>6</sup> We calculate IOR as the ratio of the number of shares held by institutions to the number of shares outstanding from CRSP. We update IOR every quarter and assume the holdings data are in the investors’ information set with a lag of one month.<sup>7</sup>

Our third sorting-variable is the *short interest ratio (SIR)*, which we calculate as short interest scaled by the number of shares outstanding (from CRSP). Short interest comes from two sources: from April 1980 to May 1988 and after June 2003, we get short-interest data from Compustat.<sup>8</sup> From June 1988 through June 2003, our short interest-data comes directly

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<sup>5</sup>Specifically, we only consider stocks with exchange code 1, 2, or 3, and share code 10 or 11. Returns are adjusted for delisting ([Shumway, 1997](#)) using the CRSP delisting return, where available. Where the delisting return is missing, we follow [Scherbina and Schlusche \(2015\)](#) and assume a delisting return of -100%, or, if the delisting code is 500, 520, 551-573, 574, 580, or 584, we assume a delisting return of -30%.

<sup>6</sup>See note issued by WRDS in May 2017. We perform some additional data cleaning. For example, we identify some firms with implausibly large jumps in IOR in a given quarter, which are generally followed by roughly equal jumps in opposite direction in the following quarter. The procedure we employ to correct such errors is described in [Appendix B.II](#).

<sup>7</sup>For example, the first trade based on December ownership data is in February of the following year. To avoid having the secular increase in IOR data coverage influence our sorts, we construct breakpoints excluding the stocks that are in CRSP but are missing ownership data. Following [Nagel \(2005\)](#), stocks with missing ownership are then assigned zero institutional ownership and consequently allocated to the low IOR portfolio.

<sup>8</sup>Compustat short-interest data are available from 1975, because we require five-years worth of data, the May 1980 portfolio is the first one that we can form. Pre-1988, Compustat short-interest data are available only for NYSE and AMEX stocks. In previous versions of the paper we started forming portfolios in June 1988, when NASDAQ data started being available. The additional eight years of data can therefore be considered an out-of-sample test. We report the main results for the subperiod of portfolios using data between 1980 and 1988 in [Appendix E.II](#). Conclusions hold. We thank a referee for making this suggestion.

from the NYSE, AMEX and NASDAQ.<sup>9</sup> However, prior to June 2003, if the short-interest data from the exchange is missing for a given firm-month, then we use the short-interest as reported by Compustat if that is available. After June 2003, if COMPUSTAT short-interest data is missing for a given firm-month, then we use the data from the exchanges if available.<sup>10</sup>

Quarterly book-equity data are from Compustat. To calculate the monthly updated *book-to-market ratio*, we divide the most recently observed book-value by the sum of the most recent market equity of all equity securities (PERMNOs) associated with the company (PERMCO). We assume that the book-value of quarter  $q$  can be observed by investors at the time of the earnings announcement for quarter  $q$ .

Our data on stock lending fees come from IHS Markit. These data start in August 2004.<sup>11</sup> We use the *indicative fee* (a proxy for marginal costs) and *simple average fee* (equal-weighted average of all contracts for a particular security) averaged within a month to assess the costs of short selling within our portfolios.

Analyst-forecasts of fiscal-year-end earnings are from IBES. We use the summary file unadjusted for stock splits, to avoid the bias induced by ex-post split adjustment, as pointed out by Diether, Malloy, and Scherbina (2002). *Forecast-dispersion (FD)* is the standard deviation of fiscal-year-end forecasts normalized by the absolute value of its mean. We eliminate values for which the mean forecast is between -0.1 and +0.1, because very low mean forecasts lead to extremely large values that bias results.<sup>12</sup>

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<sup>9</sup>We apply additional procedures to better match these short interest data with CRSP. This increases the number of firm-month observations, reduces noise and strengthens all results. Details can be found in [Appendix B.II](#).

<sup>10</sup>Exchange data from NYSE are available after September 1991, and from AMEX after 1995. Compustat data are used prior to these dates. Compustat coverage of NASDAQ is scarce before June 2003, which is why exchange data is the primary source for NASDAQ before that date. Furthermore, data from NASDAQ in February and July 1990 are missing (Hanson and Sunderam, 2014). See Curtis and Fargher (2014), Ben-David, Drake, and Roulstone (2015), and, Hwang and Liu (2014) for other papers using these data sources.

<sup>11</sup>From August 2004 Markit data frequency is weekly and daily coverage begins in July 2006.

<sup>12</sup>As an alternative specification, we follow Johnson (2004) and use total assets to normalize. Furthermore, we replicate the result with one-quarter-ahead quarterly earnings forecasts. Another alternative is to use the standard deviation in long-term growth forecasts of earnings (see, e.g., Yu, 2011; Hong and Sraer, 2016), which does not need to be scaled. Results are consistent with the findings presented in [Section 3.6](#) and can be found in [Tables F.17 to F.19](#) in [Appendix F.IX](#).

### 3 Empirical Results

In this section, we analyze the short- and long-term price dynamics of short-sale-constrained stocks following large information shocks, with the goal of characterizing the evolution of beliefs following such shocks. Once again, the idea is that the price of a constrained security will reveal the beliefs of the optimistic agents. For the constrained winners, the beliefs will be those of the agents who became most optimistic following a positive information shock in the formation period, i.e., those who overreacted to the information. By contrast, from the prices of the constrained losers, we can infer the beliefs of the agents who underreacted to the (negative) information shock in the formation period.

To identify stocks with binding short-sale constraints, we follow [Asquith, Pathak, and Ritter \(2005\)](#) and independently sort on IOR and SIR as a way of explicitly accounting for the supply- and demand-sides of the shorting market (see also [Cohen, Diether, and Malloy, 2007](#)). IOR has been shown to be closely related to lending supply (see, e.g., [D’Avolio, 2002](#)). Assuming, for example, that IOR is a direct proxy for easily available lending supply, and it is at 10%, an SIR of 10% would indicate that easily available supply is exhausted and short selling is likely constrained.<sup>13</sup> IOR and SIR are available from 1980, giving us an approximately 40-year sample with which to measure long-term returns.

Also, to identify the shocks that drive disagreement, at the start of each month  $t$ , we sort on each stock’s cumulative return from month  $t-12$  through month  $t-2$ .<sup>14</sup> For all three sorts, namely, past return, SIR, and IOR, the breakpoints are the 30th and the 70th percentile. We use independent sorts to maximize independent variation in all three variables. This  $3 \times 3 \times 3$  sort provides us with 27 portfolios. Each portfolio is value-weighted, both to avoid liquidity-related-biases associated with equal-weighted portfolios ([Asparouhova, Bessembinder, and](#)

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<sup>13</sup>This intuition is reflected in the design of the securities lending market in our model, outlined in more detail in [Appendix A.I.3](#). It is also consistent with the empirical results in [Kolasinski, Reed, and Ringgenberg \(2013\)](#) showing search frictions have a strong impact on the costs of short selling.

<sup>14</sup>We present the main results for alternative shorter momentum signals (three and six months) in [Appendix F.VII](#). Furthermore, we use abnormal returns around earnings announcements and outside of earnings announcements over the  $t-12$  through  $t-2$  window in [Appendix F.VIII](#). Results remain consistent in each of these alternative specifications.

Kalcheva, 2013) and to ensure the effects that we document are not driven by extremely low market-capitalization stocks. We label as *constrained* the set of stocks that are simultaneously in the low IOR and the high SIR portfolio. We designate the firms with the highest past returns as the *past winners* ( $W$ ), and those with the lowest past returns as the *past losers* ( $L$ ). The firms that are simultaneously constrained and either past losers or past winners are labeled as *constrained winners* ( $W^*$ ) and *constrained losers* ( $L^*$ ), respectively.

Furthermore, we want to make sure we do not confound results for our constrained loser portfolio by unintentionally blending in former constrained winners that are in the process of falling.<sup>15</sup> Therefore, we focus on the constrained losers that *were not* constrained winners within the past 5 years.<sup>16</sup> This subset of losers better reflects the return patterns of short-sale constrained stocks with initially negative news.<sup>17</sup>

### 3.1 Characteristics

[INSERT Table 1 HERE]

The left two columns of Table 1, labeled  $W^*$  and  $L^*$ , report summary statistics for the constrained winner and loser portfolios, respectively.<sup>18</sup> On average each month, 49 stocks are classified as constrained winners and 36 as constrained losers.<sup>19</sup> The representative constrained winner stock has a market capitalization of \$3 billion. Constrained losers are

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<sup>15</sup>Results for all constrained losers and the subset of constrained losers that *were* constrained winners within the past five years can be found in Appendix C.III.

<sup>16</sup>In our model, where, *ceteris paribus*, constrained winners will lose value over a number of years, they will continue to be constrained and potentially become constrained losers at some point. Such constrained losers are not losers based on a negative information shock (as in the pink profile in Panel A in Figure 2). Rather, they are former constrained winners that are already somewhere in the process of disagreement (and prices) adjusting downwards (e.g., a stock whose price behaves like the green line in Panel A of Figure 2, at period 1 or 2).

<sup>17</sup>Note the same argument does not apply the other way around, i.e., splitting the constrained winners into those that were/were not constrained losers in the past five years is not necessary. The post-formation trajectory of a constrained loser is negative initially and then flat, but never positive—so it can never be classified as a constrained winner.

<sup>18</sup>The two rightmost columns present characteristics of matched-firm portfolios, which we introduce and discuss in Section 3.3. For a comparison with the broader universe of stocks, averages for the remaining portfolios are displayed in Table C.7 in Appendix C.V.

<sup>19</sup>Note, on average, 52 individual stocks are classified as constrained losers that were constrained winners at some point in the past five years. Recall that these stocks are not included in the  $L^*$  portfolio.

considerably smaller. The formation-period returns are large in magnitude for the stocks in both portfolios: the average firm in the  $W^*$  portfolio almost doubles in size over the formation period, and the  $L^*$  portfolio almost halves.

SIR is close to IOR for the stocks in the constrained portfolios, indicating a good chance of these stocks being hard to borrow. The stocks in the constrained portfolio have low book-to-market ratios, consistent with overpricing. In addition, the constrained stocks have high idiosyncratic volatilities and high turnovers, consistent with disagreement among traders.

To check whether we accurately identify stocks with binding short-sale constraints, we calculate the level and 12-month changes of the Markit indicative and simple average loan fee over the time period for which we have these data. The fees in the constrained portfolios are large, and they have generally increased leading up to the formation date, suggesting a high and increased level of disagreement.<sup>20</sup>

Because the Markit data are only available from 2004, we calculate two additional measures for the full sample period going back to 1980. The first one is SIRIO, i.e., the number of shares sold short (short interest) divided by the number of shares held by institutions (institutional ownership), following [Drechsler and Drechsler \(2016\)](#).<sup>21</sup>

These summary statistics suggest this combination of low IOR and high SIR successfully identifies firms that are constrained. On average, the constrained winners exhibit a SIRIO of 100.31%, which likely pushes them above the point of cheap lending and makes short selling these stocks highly expensive.

For the firms and dates for which options data are available, we calculate the *option volatility spread* ([Cremers and Weinbaum, 2010](#)), defined as the difference in annualized

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<sup>20</sup>The loan fees displayed here are high, especially relative to the results in [D'Avolio \(2002\)](#), indicating short selling our constrained stocks might be prohibitively expensive. However, investors can simply benefit from the insights of this paper by avoiding past constrained winners, when running medium-/small-cap momentum strategies, as indicated by [Table D.7](#) in the Appendix.

<sup>21</sup>This measure is also attractive from the perspective of our model, because it has a direct interpretation. It tells us how close or how far above we are to the institutional-lending-supply threshold. Assuming the unknown fraction of institutions that are willing and able to lend for free is 100%, for instance, a SIRIO measure above 100% would indicate the demand for short selling is larger than the institutional lending supply, and thus investors are willing to pay high search costs in order to still be able to short the stock.

implied volatilities between at-the-money, 1–12 month-to-maturity put and call options. Note that if the volatility spread is negative, put-call parity will be violated in the sense that the price of an actual share will be higher than the price of a synthetic share (constructed using a bond, put, and call). Such a violation indicates arbitraging this put-call parity violation is costly, likely because borrowing and shorting the underlying stock is costly (Ofek, Richardson, and Whitelaw, 2004). Note both constrained portfolios have large negative volatility spreads, suggesting that shorting the underlying stocks is costly even for those engaging in this arbitrage transaction.

### 3.2 Strategy Performance over Different Horizons

Our focus is on analyzing return patterns at different horizons. The vast majority of the literature on short-sale constrained stocks examines returns at a monthly horizon (or shorter). Negative returns at this horizon could be a sign of small and temporary mispricing or of large and persistent mispricing. With our goal of understanding the longer-term belief dynamics following disagreement shocks, we examine the returns at horizons from one to five years after portfolio formation.

We start by constructing a trading strategy, like that of Jegadeesh and Titman (1993), which each month purchases \$1 worth of the new (value-weighted)  $W_t^*$  or  $L_t^*$  portfolio, holds that portfolio for the following  $T = 12$  months, and then sells it. Thus the one-year  $W^{*,1y}$  strategy at the start of month  $t$ , just after trading, consists of \$1 of the newest  $W_t^*$  portfolio, plus the last 11  $W^*$  portfolios formed in months  $t - 11$  through  $t - 1$ . In contrast to Jegadeesh and Titman (1993), we weight each of the 12 portfolios held by its cumulated dollar value, meaning we do not rebalance the invested amount for  $T$  (here,  $T = 12$ ) months.<sup>22</sup> The

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<sup>22</sup> Interpreted differently, the numerator of the buy-and-hold weight  $W$  for stock  $i$  in portfolio  $p$  at portfolio formation time  $t - 1$ , is the sum of market equity values ( $ME = PRC * SHROUT$ ) of all  $T$  occurrences at  $(t - \tau)$  this stock was allocated to portfolio  $p$  during the formation period, adjusted for the price change without dividends and adjusted for capital actions (e.g., splits, issuances or repurchases):

$$W_{i,p,t-1} = \sum_{\tau \in T} ME_{i,t-\tau} RET_{i,t-\tau,t-1}^x,$$

construction of our one-year strategy is consistent with the calendar-time portfolio approach advocated by Fama (1998). Note that we can assess the performance of these strategies with standard asset-pricing tests applied to the monthly strategy returns, and these tests will summarize the forecastable performance for the first year post-formation, much like the event-study plots in Figure 1, but without the econometric issues typically associated with studies of CARs (Barber and Lyon, 1996; Fama, 1998).

[INSERT Table 2 HERE]

The first three columns of Table 2 display summary statistics for the returns of the one-year strategies (the remaining columns present results for matched-firm strategies, which we discuss in Section 3.3). Panel A shows the raw average monthly excess returns, the number of months in the sample (T), the average number of unique stocks in the strategy in any given month (AvgN), and the realized annualized strategy Sharpe ratio (SR).

The constrained-loser ( $L^*$ ) and -winner ( $W^*$ ) strategies both have negative alphas (Panels B and C).<sup>23</sup> Indeed, the strategies both earn negative excess returns. These negative excess returns suggest that, following both positive and negative information shocks, the marginal investors (the most optimistic agents in both cases) are too optimistic initially, but revise their beliefs downwards over the first year following the information shock.

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where  $PRC$  (price),  $SHROUT$  (shares outstanding), and  $RET^x$  (ex-dividend return) are the respective CRSP variables. The weight of stock  $i$  in portfolio  $p$  consisting of stocks  $I_{p,t-1}$  is then  $w_{i,p,t-1} = \frac{W_{i,p,t-1}}{\sum_{j \in I_{p,t-1}} W_{j,p,t-1}}$ .

Traditional equal-weight calendar-time portfolios with overlapping holding-periods, as in Jegadeesh and Titman (1993), can be found in Appendix F.V. In addition, we also construct a version of the portfolios, where we just include any stock that falls into portfolio  $p$  at any point in time during the formation period (i.e., the past 12 months) weighted by the stock’s market equity at the end of the formation period  $t - 1$ . The main difference from our default buy-and-hold approach is that a stock that fell into a portfolio more than once during the past T months is only considered once here. The results of this simple value-weight specification can be found in Appendix F.VI.

Note that we obtain similar results with both the Jegadeesh and Titman (1993) and the simple value-weight specifications. A detailed discussion on measuring long-term returns can be found in Appendix B.IV.

<sup>23</sup>In Table F.3 in Appendix F.II Panels A and B, we regress 12-month buy-and-hold excess returns of  $W^*$  and  $L^*$  on a number of other well-known factors. Their returns cannot be explained by any of the factors—not even a factor that is based on the ratio of short interest to institutional ownership, as in Drechsler and Drechsler (2016). In unreported results (available upon request), we also examine the effect of using a monthly-updated version of HML as in Asness and Frazzini (2013) and characteristic efficient versions of the right-hand-side portfolios as in Daniel, Mota, Rottke, and Santos (2020). Alphas are virtually unaffected by either of the methods.

Panel B of [Figure 1](#) suggests that the predictable negative abnormal returns of the constrained-winner ( $W^*$ ) portfolios persist longer than the negative abnormal returns of the constrained-loser stocks ( $L^*$ ). To assess the statistical significance of the differences in long-term abnormal returns, we focus on years two through five post-formation. We again form strategies based on  $W^*$  and  $L^*$ , as explained above, but instead of holding portfolios formed in months  $t - 11$  to  $t$ , the strategy now holds portfolios formed in months  $t - 59$  to  $t - 12$ ; that is, we skip the most recent year and hold 48 portfolios from the preceding four years.<sup>24</sup>

[INSERT [Table 3](#) HERE]

[Table 3](#) presents the results. The average number of unique stocks held in the strategy is quite large now: specifically, 417 unique stocks were classified as constrained winners in at least one month between 2 and 5 years prior to strategy formation.<sup>25</sup> Panel A presents raw excess returns and Sharpe ratios of the strategies. Strikingly, whereas the constrained-loser strategy does not exhibit a significantly negative alpha, the constrained-winner strategy significantly underperforms relative to the Fama-French-Carhart model, with an alpha of  $-0.62$  and a  $t$ -statistic of  $-4.93$ .<sup>26</sup> The difference in abnormal returns between  $W^*$  and  $L^*$  is  $-0.88\%$  per month with a  $t$ -statistic of  $-4.33$ .<sup>27</sup>

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<sup>24</sup>Each month, the most recent (12-month-old) constrained portfolio is added with \$1, and then, no adjustment is made to the investment amount for the remaining 48 months of holding. The first holding month is April 1990, that is, the first time we are able to determine portfolio membership for 48 months in a row.

<sup>25</sup>[Figures C.2 to C.4](#) show time variation in the number of stocks in the portfolios. The  $W^*$  portfolio of years two through five ([Figure C.4](#)) contains between 200 and more than 700 stocks. The  $L^*$  portfolio contains between 100 and 400 stocks.

<sup>26</sup>A 60-month buy-and-hold portfolio of constrained winners, that does not skip the first 12 months after formation, yields a four-factor information ratio of  $-0.73$  (see [Appendix C.III Table C.4](#)). Such a portfolio has 511 unique stocks in it. Moreover, using the simple value-weight approach, described in [Footnote 22](#), a strategy using allocation between months  $t - 60$  and  $t - 1$  generates a four-factor information ratio of  $-1.02$ .

<sup>27</sup>In [Table C.3](#) in [Appendix C.III](#), we contrast  $L^*$  with a portfolio of constrained losers that *were* constrained winners in the five years before they became constrained losers ( $L^{*W}$ ).  $L^{*W}$  has a negative (albeit insignificant) alpha. The difference between the two is significant at the 5% level. Moreover, spanning tests, reported in [Table F.4](#), show constrained winners help explain the long-run returns of *all* constrained losers, whereas the opposite is not true. The result holds for raw returns as well as when the the three [Fama and French \(1993\)](#) factors and momentum are included. Both of these results are consistent with the  $L^{*W}$  stocks (i.e., those constrained losers that were constrained winners within the past five years) driving the low long-run returns of the combined constrained-loser portfolio ( $L^{*all}$ ).

These results suggest that, following a positive-information shock, the most optimistic agents continue to revise their beliefs downwards in years two through five post-formation. By contrast, following a negative-information shock, we find no evidence that the optimistic agents continue to revise their beliefs downwards after one year; the beliefs of the optimistic agents are relatively stable after about one year.

As a robustness check, we use the [Fama and MacBeth \(1973\)](#) procedure in [Appendix F.I](#), which allows us to control for various other measures and generates consistent results.

[INSERT [Figure 3](#) HERE]

In Panel A of [Figure 3](#), we plot the time series of the cumulative one-year strategy returns for the  $W^*$  and  $L^*$  portfolios, hedged with respect to the CAPM-Mkt factor over the sample period.<sup>28</sup> Both strategies fall consistently over the full sample period, suggesting the strategy underperformance is not concentrated in a particular subperiod. The magnitude of the underperformance results in a dramatic negative cumulative return: an initial investment of \$1,000 into the one-year hedged past-winner (-loser) strategy is worth \$18.90 (\$0.34) at the end of June 2020.

For comparison, Panel B plots the cumulative returns for the two to five-year strategies for constrained winners and losers. Here the difference between the two strategies again does not appear to be driven by a particular subsample.

[Appendix E.I](#) contains a replication of our main results on a sample of international equities, and [Appendix E.II](#) presents results for a subsample using data before short interest was available for NASDAQ stocks.

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<sup>28</sup>Specifically, we calculate the returns to the portfolios for each sample month. We then run a full-sample regression of the portfolio excess returns on Mkt-RF. Then, using the full-sample regression coefficient, we subtract the returns of the zero-investment hedge portfolio [ $b_{\text{Mkt}}(R_{\text{Mkt}}-R_{f,t})$ ] from the respective portfolio excess returns to generate the hedged excess returns. The factor return data come from Kenneth French's data library.

### 3.3 Matching

In the preceding subsection, we document that constrained past winners and losers exhibit strongly predictable negative abnormal returns, and that the persistence of these negative returns is about five years for the constrained winners, but only one year for the constrained losers. We hypothesize that this asymmetric return predictability results from a combination of short-sale constraints and differential interpretation of information shocks. We argue our portfolios are constrained because of a combination of low IOR and high SIR. Specifically, the low IOR is a proxy for a low lending supply (because non-institutional holders are less likely to make their shares available for lending). Then, when disagreement and strong demand to borrow and (short) sell shares from pessimistic investors arise, short interest will be high, reflecting a demand for borrowing that exceeds the supply of easily borrowed shares, and resulting in borrowing becoming costly. Thus, we use a combination of high-short interest and low institutional ownership as a proxy for the combination of low lending supply and high borrowing demand that we label as *constrained*.

However, another possibility is that, in sorting on IOR, SIR, and past return to form our portfolios, we are inadvertently selecting some other firm characteristic that is directly linked to return predictability. For example, previous literature argues short interest is a proxy for informed demand and thus predicts future returns (Boehmer, Jones, and Zhang, 2008; Boehmer, Huszar, and Jordan, 2010; Engelberg, Reed, and Ringgenberg, 2012; Rapach, Ringgenberg, and Zhou, 2016). Sorting on high short interest alone, in combination with high or low past return, could give the same results.

To examine this possibility, for each constrained portfolio, we create a matched portfolio that contains firms that are as close to identical as possible to the firms in our constrained portfolio on the dimensions of size, short interest, past return, and the natural logarithm of book-to-market, except these matched firms are *not* short-sale constrained (i.e., they have high institutional ownership). For each stock in the constrained-winner portfolio ( $W^*$ ) and the constrained-loser portfolio ( $L^*$ ), we run a matching procedure based on the Mahalanobis

(1936) distance to find a statistical twin stock in a universe of unconstrained potential matches. We limit the unconstrained matching universe to stocks above the 70th percentile cutoff for institutional ownership, and to fall within the same past return bucket. For the losers, we also impose the constraint that the matched firm was not a constrained-winner stock at any time within the past five years. We then identify, in each month and for each firm in one of the portfolios, the nearest neighbor based on the Mahalanobis distance considering the four matching dimensions: size, short interest, past return, and log book-to-market.<sup>29</sup>

The last two columns in Table 1 reveal the value-weighted portfolios of matched stocks for  $W^*$  and  $L^*$ , that is,  $W_m^*$  and  $L_m^*$ , are similar along the matching dimensions of size, short interest, past return, and book-to-market and on related dimensions (e.g., turnover and volatility). However, they differ substantially on dimensions that proxy for short-sale constraints, such as SIRIO, volatility spread, and Markit loan fees. This finding suggests the matched firm-portfolios should be well suited to uncovering differences that are based solely on the fact that one set of firms is short-sale constrained while the other is not.

Table 2 shows that one-year strategies based on matched firms exhibit no abnormal performance relative to a CAPM or four-factor model. However, the final three columns show the strategies based on constrained winners and losers both dramatically underperform their matched-firm strategies. The final column of this table shows the difference-in-differences ( $DiD$ ) is statistically indistinguishable from zero.

However, the picture changes for the following four years (Table 3): constrained winners significantly underperform unconstrained ones, whereas constrained and unconstrained losers exhibit very similar returns. The  $DiD$  is now highly statistically significant,<sup>30</sup> even when

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<sup>29</sup>We thank our discussant Adam Reed for suggesting the use of short interest as a matching variable. In Appendix F.III, we redo this exercise with an alternative set of matching variables, namely, size, book-to-market, past return, and institutional ownership, making short interest the variable that distinguishes constrained from matched stocks. The results remain large and statistically significant for this specification. Using past long-term return (months t-36 through t-13) instead of the log book-to-market ratio as a matching variable also has little impact on the results (available upon request). Matching based on propensity scores instead of Mahalanobis distances produces very similar results as well (available upon request).

<sup>30</sup>Standard errors are adjusted for heteroskedasticity and serial correlation using the Newey and West (1987) procedure, where the number of lags is determined automatically (Newey and West, 1994). The detected lags vary between 8 and 11 (except for  $L^*$ , where it is 2). Furthermore, standard errors are slightly

controlling for the market (Panel B) or the four Fama-French-Carhart factors (Panel C). Consequently, we reject the set of hypotheses postulating that our matching variables are responsible for the observed asymmetry in mispricing persistence.

[INSERT Figure 4 HERE]

Figure 4 adds more background to the matching approach. It shows the buy-and-hold performance of the constrained and matched portfolios on a year-by-year basis.<sup>31</sup> Whereas the constrained stocks show distinct and significant patterns of mispricing, the matched portfolios' returns can always be explained by the CAPM.<sup>32</sup> Note constrained winners underperform significantly relative to the CAPM for *each* of the five years following formation (Panel A), whereas constrained losers only exhibit a significantly negative alpha in the first year (Panel B). This visual assessment is consistent with the time-series regressions presented throughout the paper.

### 3.4 Earnings Announcements

One point in time when disagreement is likely to be resolved is when firms announce their earnings (see, e.g., Berkman, Dimitrov, Jain, Koch, and Tice, 2009), which usually happens once per quarter.<sup>33</sup> Disagreement-based explanations of the performance of constrained stocks predict negative abnormal returns are concentrated in times of decreasing disagreement.

[INSERT Figure 5 HERE]

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smaller than White standard errors, indicating negative residual autocorrelation. Statistical inference for our main results based on various bootstrapping techniques can be found in Appendix B.III.

<sup>31</sup>Specifically, we calculate the buy-and-hold return, as explained in Section 3.2 for the first holding-year, for each following year, in the same fashion. We then run a time-series regression of the monthly excess returns of these 12-month buy-and-hold portfolios on the CAPM-Mkt factor. The annualized alpha as well as the 95% confidence interval, constructed based on Newey-West standard errors, are plotted for each year after formation.

<sup>32</sup>Note we cannot reject the hypotheses that the average first-year returns of the matched winners and losers in Figure 4 are equal to those of the winners and losers in the full sample plotted in Figure 1 Panel A and tabulated in Table D.4. The  $p$ -values from Welch-two-sample- $t$ -tests are  $p = 0.67$  for winners and  $p = 0.55$  for losers.

<sup>33</sup>We directly test the assumption that earnings announcements decrease disagreement in Appendix D.I by looking at dispersion in beliefs before and after announcements. Disagreement goes down significantly for firms with high initial disagreement, consistent with our assumption.

Figure 5 displays average cumulative abnormal returns (ACAR) of constrained winners and losers around earnings announcements, for stocks selected to one of the portfolios in the previous year (Panel A) or the four years preceding that year (Panel B). Daily abnormal return is defined as the return adjusted for the CAPM-MktRF factor.<sup>34</sup> Constrained winners and losers fall considerably on the first five days following the announcement for stocks selected in the preceding 12 months, and continue to underperform thereafter (Panel A). For stocks for which the portfolio allocation dates back more than a year, a much stronger reaction can be observed for winners than for losers. Moreover, the pre-announcement rise is larger than the post-announcement drop for losers in Panel B.

### 3.5 Lending Fees

Whereas the empirical analyses of portfolio returns of constrained stocks reveal the beliefs of the most optimistic agents, costs associated with short selling stocks can reveal information about the pessimists' beliefs. Here, we use the Markit indicative lending fee as a proxy for the marginal costs of short selling. This lending fee clearly will not capture all of the costs or risks associated with short selling, and therefore may not be a good proxy for the level of the costs associated with short selling. However, changes in the fee over time should be correlated with changes in the pessimistic agents' willingness to pay to borrow shares for the purposes of shorting, and therefore with changes in disagreement between pessimists and optimists for constrained stocks.

Panel C in Figure 1 reveals that the fees increase leading up to portfolio formation and then fall slowly over about five years post-formation for both constrained winners and losers. On the surface, this finding might seem puzzling, because price predictability is asymmetric for winners and losers, whereas disagreement apparently is symmetric. However, it is completely consistent with the example presented in the Introduction and with the model in Section 4.

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<sup>34</sup>The calculation of abnormal returns is explained in detail in Appendix B.V.

[INSERT Table 4 HERE]

The results can be confirmed by regressing changes in portfolio lending fees on dummy variables that equal 1 either when an observation corresponds to the first year post-formation ( $I_{0 < k \leq 12}$ ), years two through five post-formation ( $I_{12 < k \leq 60}$ ) or later ( $I_{k > 60 \leq 72}$ ), displayed in Table 4. Disagreement significantly falls in the first year and years two through five, for both constrained winners and constrained losers. No significant difference exists in their decay rates. For winners, the fee continues falling even after year five, although the absolute slope is decreasing and the coefficient estimate’s standard error is much larger than that of the earlier years.

### 3.6 Analyst Disagreement

Thus far, we have inferred beliefs and disagreement from market prices. An alternative is to look at forecasts from analysts. An advantage is that such forecasts are direct measures of beliefs. However, some doubts have emerged regarding whether these data might contain systematic biases related to the incentive structure in that industry and, moreover, whether they represent the marginal investor (Hong, Kubik, and Solomon, 2000; Lim, 2001; Hong and Kubik, 2003). With these caveats in mind, these data nonetheless allow us to test belief dynamics out-of-sample.<sup>35</sup> Hence, we generate some stylized empirical facts about the dynamics of the disagreement process of analysts in this section.

To analyze the dynamics of analyst beliefs, we first sort stocks into five portfolios based on the preceding year’s change in dispersion,  $\Delta FD$ .<sup>36</sup> Figure 6 plots forecast dispersion from one year before until five years after portfolio formation. The high- $\Delta FD$  portfolio reverses within roughly five years, consistent with the resolution of disagreement taking about five years. The timing is also consistent with the return patterns we observe, where constrained

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<sup>35</sup>Another caveat with these data is that they are only available for the largest stocks for which we typically do not observe binding short-sale constraints. For example, only 20% of the stock-month observations that we identify as having low IOR (i.e., in the bottom 30%) have a non-missing forecast dispersion. Hence, to study returns, in the previous sections, we resort to the proxy based on past return and SIR.

<sup>36</sup>Average characteristics of these portfolios can be found in Table C.9 in Appendix C.VII.

winners lose significantly in value for roughly five years. The second-highest-change portfolio already exhibits a much lower increase in disagreement, indicating large changes are rare. A small predictability in the other direction seems to exist, as the low- $\Delta FD$  portfolio slightly bounces up after portfolio formation. This increase is small in magnitude relative to the predictability of the high-change portfolio, though, and the level arrives nowhere near its previous high. In addition to these  $\Delta FD$ -sorted portfolios, we examine the same  $FD$  measure for the intersection of the high- $\Delta FD$  and momentum portfolios in [Figure 1](#) Panel D. We observe a symmetric pattern for winners and losers, consistent with the evidence from lending fees.

[INSERT [Figure 6](#) HERE]

In [Table 5](#), we predict future changes in  $FD$  over the next year (columns 1 and 2), the following four years (columns 3 and 4), and over five years (columns 5 and 6) with positive and (the absolute value of) negative changes in analyst disagreement over the past year ( $\Delta FD_{(t-12)-t}^+$  and  $|\Delta FD_{(t-12)-t}^-|$ ), using the market-capitalization-weighted [Fama and MacBeth \(1973\)](#) procedure. The results confirm that positive past changes strongly predict negative future changes both for one year and the following four years ( $t+12 - t+60$ ). By contrast, including negative past changes in the regression barely increases the time-series average of the cross-sectional  $R^2$ , and, despite being statistically significant for the first year, the coefficient estimate for negative past changes is smaller by an order of magnitude in the first year than that of the positive past changes.

[INSERT [Table 5](#) HERE]

We conclude that the dynamics of analyst disagreement are consistent with the results from our analysis of lending fees in that, following large shocks to disagreement, disagreement falls over the course of about five years. Moreover, this decay is symmetric for securities that experienced either positive- or negative-information shocks. This result is broadly consistent with the evidence in [Yu \(2011\)](#), who finds aggregate market disagreement is slowly mean-reverting (with a half-life of about one year).

## 4 Theoretical Implications

In this section, we discuss theoretical implications of the empirical work presented in [Section 3](#), specifically on how beliefs and disagreement evolve following information shocks. These findings have a natural link to existing theories of momentum and long-term reversals, and we discuss the implications for these theories in [Section 4.1](#). We conclude our empirical findings are inconsistent with existing explanations of momentum. In [Section 4.2](#), we develop a new model that is consistent with both our new empirical findings and with extant evidence on the behavior of unconstrained stocks.

### 4.1 Existing Explanations of Momentum and Reversals

Rational models of momentum posit that the premium earned by high momentum is compensation for risk, either because of high unconditional exposure to priced factors (see, e.g., [Berk, Green, and Naik, 1999](#); [Johnson, 2002](#); [Sagi and Seasholes, 2007](#)), or because momentum serves as a proxy for time-varying exposure to factors with a time-varying premium ([Kelly, Moskowitz, and Pruitt, 2021](#)). These arguments are hard to reconcile with short-term losses and asymmetric long-term predictability among constrained stocks that we document here. Constrained winners and losers must have substantially different time-varying exposures to the priced risk factors, which are not present in a similar way among unconstrained stocks. In sum, rational explanations do not easily lead to a joint explanation of the impulse-response functions following large price shocks for constrained and unconstrained stocks.

Alternative starting points are the existing behavioral explanations offered for momentum and long-term reversal among unconstrained stocks, such as [Daniel, Hirshleifer, and Subrahmanyam \(1998\)](#), [Barberis, Shleifer, and Vishny \(1998\)](#), and [Hong and Stein \(1999\)](#), combined with the explanation for the underperformance of short-sale constrained stocks in the presence of disagreement offered in [Miller \(1977\)](#). Miller argues that when disagreement exists about the value of a security and when pessimists are constrained from short selling,

the security price will reflect only the views of the optimists. When disagreement is resolved over time, this overvaluation is eliminated, leading to predictably low returns.

However, naive combinations of the Miller (1977) model with any of these three models yields predictions that are inconsistent with empirical findings: Daniel, Hirshleifer, and Subrahmanyam (1998) and Barberis, Shleifer, and Vishny (1998) are both representative-agent models with no disagreement. Without disagreement, introducing short-sale constraints would have no effect on prices and thus could not generate the return patterns we document here. Hong and Stein (1999) is a heterogeneous-agents model, but the nature of the *newswatcher* and *momentum-trader* belief-formation process in their model implies that even in the presence of short-sale constraints, expected returns would still be positive for past winners, which is inconsistent with the empirical evidence we present here.<sup>37</sup>

In sum, none of the approaches above are able to offer a satisfying explanation of the strong asymmetry in mispricing persistence that we observe between constrained winners and losers.

## 4.2 A Model of the Dynamics of Disagreement

We now propose a model that combines key features of models from the literature reviewed in Section 4.1 in a parsimonious way. Here, we present an overview of the model and provide the intuition for the key implications. Appendix A provides a formal development of the model.

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<sup>37</sup>An ad-hoc, empirically motivated explanation for the short-term patterns is what we call the *additive-effect hypothesis*: suppose momentum and long-term reversal effects exist for all stocks. Further suppose constrained stocks generally underperform and that the constrained-stock effect is roughly as large as the long-term reversal effect at longer horizons. To reflect the effect of short-sale constraints, we could simply add a constant negative slope to the impulse-response function of both unconstrained winners and losers and assume this slope just cancels out with the positive long-term reversal of past losers (i.e., the magnitudes of both effects are about the same). This hypothesis would predict long-term mispricing persistence for winners but not for losers, in addition to no winner momentum and amplified loser momentum in the short run. A testable implication of the additive-effect hypothesis is that the *DiD* between constrained winners and unconstrained winners and constrained losers and unconstrained losers is the same at all horizons. The effect of short-sale constraints is always what is added to the empirical patterns of momentum and long-term reversals for winners and losers. In Section 3.3, we employ a matching approach to specifically test this. The data clearly reject the hypothesis: constrained winners underperform, whereas constrained losers exhibit no significant difference, relative to their matched counterparts, in years two through five post-formation. The *DiD* is highly statistically significant (see Table 3 in Section 3.3).

Specifically, we combine overconfidence and slow information diffusion in a model that implies the belief dynamics consistent with the empirical findings we documented in [Section 3](#). Our model is in the tradition of other models that formalize the idea that differences of opinion combined with short-sale constraints will influence asset prices (see, e.g., [Harrison and Kreps, 1978](#); [Diamond and Verrecchia, 1987](#); [Duffie, 1996](#); [Chen, Hong, and Stein, 2002](#); [Hong and Stein, 2003](#); [Scheinkman and Xiong, 2003](#); [Gallmeyer and Hollifield, 2007](#); [Ang, Shtauber, and Tetlock, 2013](#); [Hong and Sraer, 2016](#)).

[Duffie, Gârleanu, and Pedersen \(2002\)](#) explicitly model the complex search and matching process on the lending market. We instead model the lending market as a market in which lending supply and demand determine equilibrium quantities in the same way as for a stock, or in a standard goods market (see, e.g., [Blocher, Reed, and Van Wesep, 2013](#)).

The model features heterogeneous risk-averse agents with differing beliefs about a single asset’s cash-flows. The agents have CARA preferences and the asset’s payoff is conditionally normally distributed. In this setting, the equilibrium price is a linear function of the investors’ beliefs in the absence of frictions (see, e.g., the discussion of the competitive equilibrium in Chapter 12 of [Campbell, 2018](#)). However, short-sale costs can partly or fully sideline some of these agents, leading to a different equilibrium price that no longer reflects the beliefs of the more pessimistic market participants.<sup>38</sup>

In our setup, heterogeneous agents with CARA trade an asset that will pay a liquidating dividend in period  $T$  that is the sum of dividend innovations in each period  $t = 1, \dots, T$ . Agents disagree about the mean and the variance of these dividend innovations, but as these agents observe the innovations each period, they update their priors.

For modeling convenience, we follow recent behavioral models (see, e.g., [Barberis, Greenwood, Jin, and Shleifer, 2018](#); [Da, Huang, and Jin, 2021](#)) in assuming that in each period  $t$ , each agent maximizes her expected utility over period  $t + 1$  wealth. To solve this port-

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<sup>38</sup>By “sideline,” we mean the agent would choose to short the security in the absence of the costs of borrowing. Agents may be partly sidelined, in the sense that they short less of the security than they otherwise would, or fully sidelined, in the sense that they choose not to participate at all (i.e., to short zero shares).

folio optimization, each agent needs to forecast the distribution of the equilibrium price in period  $t + 1$ , which will be based on the agents' beliefs at that time. We assume that, in calculating this distribution, each agent makes the strong assumption that disagreement will be resolved in the following period in such a way that each other agent will come to agree with her. This assumption makes the solution far more tractable and is consistent with [Kahneman and Tversky \(1973\)](#)'s "illusion of validity."<sup>39</sup> In other words, agents believe their views are correct and that others will agree with them sooner rather than later.

A model feature that drives our results is that access to private information is paired with overconfidence. Motivated by this feature, our model has two types of agents. The first set of agents are informed and *overconfident*. They receive all new information immediately. Consistent with [Daniel, Hirshleifer, and Subrahmanyam \(1998\)](#) and [Gervais and Odean \(2001\)](#), this access to information makes them overconfident about the signal they receive, in that they assess the precision of their signal to be higher than it actually is.<sup>40</sup>

The second set of agents—whom we label *newswatchers*—are similar to the newswatchers of [Hong and Stein \(1999\)](#) in that the new information (that the informed overconfident agents observe immediately) slowly diffuses through the population of newswatchers. Crucially, we follow [Hong and Stein \(1999\)](#) in assuming newswatchers ignore the information content of prices; that is, they fail to infer informed agents' signals from prices (also see [Luo, Subrahmanyam, and Titman, 2020](#), where traders are skeptical about the ability of others). [Hong and Stein \(1999\)](#) put forth slow information diffusion as an explanation of shorter-term momentum effects, whereas [Daniel, Hirshleifer, and Subrahmanyam \(1998\)](#) use

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<sup>39</sup>[Kahneman and Tversky \(1973\)](#) suggest the term "illusion of validity" for the observation that "people are prone to experience much confidence in highly fallible judgments." [Kahneman \(2011\)](#) links this illusion to the financial industry (see pages 212 to 216 for a discussion on what Kahneman calls "the illusion of stock-picking skills").

<sup>40</sup>The behavioral sciences also provide evidence that overconfidence can increase with more information. [Oskamp \(1965\)](#) asked psychologists to assess the personality of a patient. Information was provided in chunks, and judges responded to a questionnaire after having received a new piece of information. With more information, judges believed they became more accurate in assessing the personality of the patient, although their predictive ability actually stayed constant. Applied to financial decision-making, this result implies overconfidence should be more pronounced among professionals than among novices. In a study by [Glaser, Langer, and Weber \(2013\)](#), traders indeed showed a higher degree of overconfidence than students in two forecasting tasks.

the resolution of overconfidence to explain longer-term reversal effects. Consistent with these papers, we assume that the resolution of overconfidence requires more time than the information-diffusion process,<sup>41</sup> and show the interaction of newswatchers and overconfident agents generates standard short-term momentum and long-term reversal effects for unconstrained stocks.

The intuition for the key model implications is as follows: First, consider an unconstrained stock for which news about the asset's cashflows is strongly positive. This information is first observed by the informed (and overconfident) agents who, by virtue of their overconfidence, put too much weight on the information. By contrast, newswatchers do not initially receive this information and do not infer the existence of the information from prices. Thus, in response to this information shock, the price moves up as overconfident agents buy and newswatchers sell. Moreover, as the new information diffuses through the population of newswatchers, the price increases further, generating momentum. Once the information has completely diffused through the population, the price will no longer rise. The fact that momentum lasts about a year suggests the information diffusion should also last about a year. After a year, the price will start to decline as more information is released and the overreaction of the informed overconfident agents is corrected, producing a long-term reversal effect. For unconstrained stocks, the momentum/reversal effect will be symmetric for positive- or negative-information releases. However, the effect will not be symmetric for constrained stocks.

By contrast, a heavily constrained stock that experiences a similar information shock will also increase in price in response to the informed overconfident agents seeing this new information. However, the newswatchers will now be sidelined. This implies price dynamics largely follow the belief dynamics of overconfident agents and these firms quickly become

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<sup>41</sup>This assumption is also consistent with the evidence reported in [Boutros, Ben-David, Graham, Harvey, and Payne \(2020\)](#), who show strong-conviction/overconfident miscalibration is extremely persistent. Conformity bias ([Wason, 1960](#); [Klayman and Ha, 1987](#); [Rabin and Schrag, 1999](#)) provides a plausible mechanism for why overconfident people are slow in revising their priors. It is also consistent with the long persistence of analyst disagreement presented in [Section 3.6](#).

overpriced. The resolution of overconfidence takes as long as for unconstrained stocks, resulting in low long-term returns for these stocks.

For constrained firms that become “losers” as a result of bad news about cash flows, the overconfident agents are the ones who will be sidelined, because they are the most pessimistic, and the newswatchers will therefore set prices. These loser stocks are overpriced as well, as the negative information diffuses slowly into the price. However, in contrast to constrained winners, strong negative returns of constrained losers will only be observed over the shorter time period over which information diffuses.

Thus, our model, which produces standard momentum and long-term reversal effects for unconstrained stocks, suggests that for constrained stocks, no momentum effect arises for winners, but losers experience an exaggerated momentum effect. Our model further suggests both constrained winners and constrained losers earn strong negative future returns. An interesting implication of our model is that for the past loser firms, overpricing will be eliminated over the short horizon over which momentum is observed, namely, about one year. For the past winner firms, the elimination of overpricing will take as long as long-term reversal effects, namely, about five years. These predictions are consistent with the empirical findings documented in [Section 3](#).

To illustrate the connection of the model with the empirical results discussed in the previous parts of the paper, consider winners and losers for two extreme cases: either a stock can be shorted without any costs (unconstrained) or a stock cannot be shorted at all (constrained). Panel B of [Figure 2](#) shows the evolution of posterior beliefs over time  $t$  of overconfident agents (labelled “Overreacting agent”)  $\mathbb{E}_{O_t}[D_T]$  and newswatchers (“labelled underreacting agent”)  $\mathbb{E}_{N_t}[D_T]$  about the liquidating dividend  $D_T$ , as well as the rational-expectations beliefs of a Bayesian who sees the dividend innovations of the overconfident agents. Focus on the case in which the price initially shoots up; that is the firm experiences a large positive dividend innovation, “good news”, in the first period. Overconfident agents immediately observe the new information/signal, overestimate its precision and therefore

overreact to it, and thus become far too optimistic about the value of the final liquidating dividend  $D_T$ . Over time, the overconfident agents learn (slowly) from further dividend innovations and converge towards the Bayesian price expectation. By contrast, the newswatchers take three periods to see all the positive information that the overconfident agents see in the first period. However, they do not overreact, and, as a consequence, their belief step-wise approaches the rational-expectations belief. In period  $t = 2$ , beliefs of newswatchers and rational-expectations beliefs finally coincide.

What are the consequences for asset prices? For unconstrained assets (unconstrained winners in Panel A), our heterogeneous-agents model states that the equilibrium price is simply a weighted average of agents' beliefs. As a consequence, and given the beliefs of overconfident agents and newswatchers, the asset price in an unconstrained market, the dashed blue line in Panel A of [Figure 2](#), is the weighted average of the beliefs shown in Panel A. Overconfident agents are long, whereas newswatchers are short in the stock. The price path exhibits short-term momentum caused by slow information diffusion among the newswatchers (as in [Hong and Stein, 1999](#)). After newswatchers have learned the Bayesian expectation, the stock is overpriced, because overconfident agents are still too optimistic (as in [Daniel, Hirshleifer, and Subrahmanyam, 1998](#)) about the final liquidating dividend  $D_T$ . The overpricing vanishes in the long run, consistent with long-term reversal effects.<sup>42</sup>

The dynamics of prices are fundamentally different for a *constrained* winner. The opinions of the newswatchers, who become pessimists following positive new information shocks, are now completely sidelined from the market. This results in the price reflecting just the beliefs of the overconfident agents. As a consequence, the price overshoots in response to the large dividend innovation in period  $t = 1$ . We do not see a momentum effect. The source of the momentum effect, slow information diffusion, plays no role in the price-setting process,

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<sup>42</sup>Note we have deliberately chosen a calibration of our model that predicts a short-term momentum and a long-term reversal effect for unconstrained stocks. Choosing an extreme parameterization, where no such effects exist, is possible. However, such calibrations are clearly inconsistent with the large empirical evidence on momentum and long-term reversal for unconstrained stocks.

because the market price no longer reflects the newswatchers' beliefs. The stock experiences long-term negative returns caused by the slow resolution of overconfidence.

Figure 2 also shows the beliefs and prices for a firm that experiences a negative-information shock (a loser stock). The assumptions of our example are unchanged, except that all information is multiplied by  $-1$ . Beliefs in Panel B and the price dynamics for the unconstrained loser (Panel A) mirror the beliefs and price dynamics of an unconstrained winner. The overconfident agents, who overreact to the large negative surprise in the initial period, are now the pessimists and short the stock. Short-term momentum is again caused by slow information diffusion, and long-term reversal has its roots in the resolution of overconfidence over time.

When firms become short-sale constrained, this symmetry between winners and losers breaks down. The high cost of short selling sidelines the pessimists who are now, in the case of negative news, the overconfident agents. Prices therefore reflect the newswatchers' beliefs (Panel A, lower half). This results in an exaggerated momentum effect, as prices in periods 0 and 1 are higher than they would be in the unconstrained case. After the newswatchers have seen all the negative information, no long-term reversal effect occurs. The opinions of pessimistic overconfident agents, who are causing the long-term reversal effect in the unconstrained case, are still sidelined from the market valuation.<sup>43</sup>

We conclude this discussion with two remarks: First, the beliefs of the informed overconfident agents and the newswatchers in our model match the belief patterns of optimists for constrained stocks that the empirical analysis has suggested. In principle, behavioral assumptions of each group can be substituted with alternative assumptions that imply similar dynamics of beliefs. Agents in these groups may suffer from different biases or face information structures currently unexplored in our model. For example, in an alternative model with the right assumptions, return extrapolator beliefs could be consistent with those of

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<sup>43</sup>Note that in a setting where short selling is costly but not impossible, we would see a long-term reversal effect for constrained losers. However, the effect would be smaller than in the unconstrained case, because the beliefs of overconfident agents would be partly sidelined.

the informed overconfident agents in this model. We do not attempt to distinguish between competing or complementary assumptions. Matching these belief dynamics likely requires that at least two groups of agents exist: one group that overreacts and whose miscalibration is highly persistent, and a second group that underreacts over a shorter horizon.

Second, agents within a group have homogeneous preferences in our model. A useful extension could introduce wavering (Barberis, Greenwood, Jin, and Shleifer, 2018), that is, the idea that agents' beliefs are weighted averages of Bayesian and non-Bayesian beliefs. Agents within a group have similar, but not the same weights. These weights fluctuate over time, causing disagreement within a group. As a result, large price shocks that drive large wedges between Bayesian and non-Bayesian beliefs, lead to within-group disagreement, and the larger the shock, the greater the disagreement. This extension could explain high turnover during the formation period of momentum stocks.<sup>44</sup> To the extent that overconfident investors hold more extreme non-Bayesian beliefs and are generally more prone to wavering, turnover for constrained winners should increase more than turnover for constrained losers (see Table 1 for evidence broadly consistent with this prediction).

## 5 Conclusion

We document a strong asymmetry in mispricing persistence between constrained winners and constrained losers. Constrained past losers exhibit no abnormal returns one year after portfolio formation, but constrained past winners continue to underperform for another four years. The overpricing of the portfolio of constrained winners is economically large: this portfolio loses more than 50% relative to the market in the five years following portfolio formation. Proxies for disagreement, such as lending fees or analyst forecast dispersion indicate that the beliefs of pessimists and optimists decay slowly over about five years.

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<sup>44</sup>See Barberis, Greenwood, Jin, and Shleifer (2018) for more details on wavering and trading volume, including empirical evidence on mutual-fund and hedge-fund holdings during the dotcom era. Broadly consistent with these ideas, Cookson and Niessner (2020) show empirically that disagreement among agents with the same investment approach is related to trading volume.

Strikingly, we find no difference in disagreement persistence between constrained winners and losers, nor between unconstrained winners and losers.

Our explanation for the asymmetry in mispricing persistence and the symmetry in disagreement persistence relies on the intuition developed in [Miller \(1977\)](#): the price dynamics for constrained stocks mirror the beliefs of optimists. The prices of constrained winners reflect the optimism of overreacting agents, who slowly revise their beliefs, leading to persistent negative returns. The strong but less persistent negative returns for constrained losers suggest optimism for these stocks disappears quickly. These facts, combined with the high persistence in disagreement for both positive and negative shocks, is consistent with the existence of two groups of agents whose beliefs evolve in different ways in response to these shocks: one group overreacts to this new information, and the bias associated with this overreaction remains strong for out to five years. Another group underreacts, but this underreaction is corrected within about a year.

We propose a model grounded in documented psychological regularities that explains these dynamics based on overconfidence and slow information diffusion. The agents with immediate access to the new information place too much weight on this new information as a result of overestimating its precision. Over time, as new information is received, these agents necessarily start to put less weight on the initial signal, but this is a slow process, so the overreaction is persistent. The agents who do not immediately observe the full signal do not overestimate its precision. However, they only observe the full signal with a lag as a result of slow information diffusion. This model generates beliefs consistent with the price and disagreement dynamics observed for the constrained and the unconstrained stocks. However, we remain circumspect on the question of whether it is necessarily this set of mechanisms that is responsible for the dynamics of disagreement we observe.

For future research, our analysis suggests short-sale constrained stocks can be used as a unique testing ground for heterogeneous-agents models, because their predictions for constrained and unconstrained assets will typically differ when some agents are sidelined from

the market. The prices of constrained stocks when combined with data on disagreement can provide a new window into the distribution of beliefs, and thus can help us understand how prices are set for unconstrained assets. Our empirical analysis provides an example of this idea. The empirical price patterns of constrained stocks following large price shocks suggest a belief-formation process that is inconsistent with leading models of momentum and long-term reversals. To the extent that belief-formation processes for unconstrained and constrained stocks are similar, our results suggest either that existing explanations are incomplete or that fundamentally different explanations are needed.

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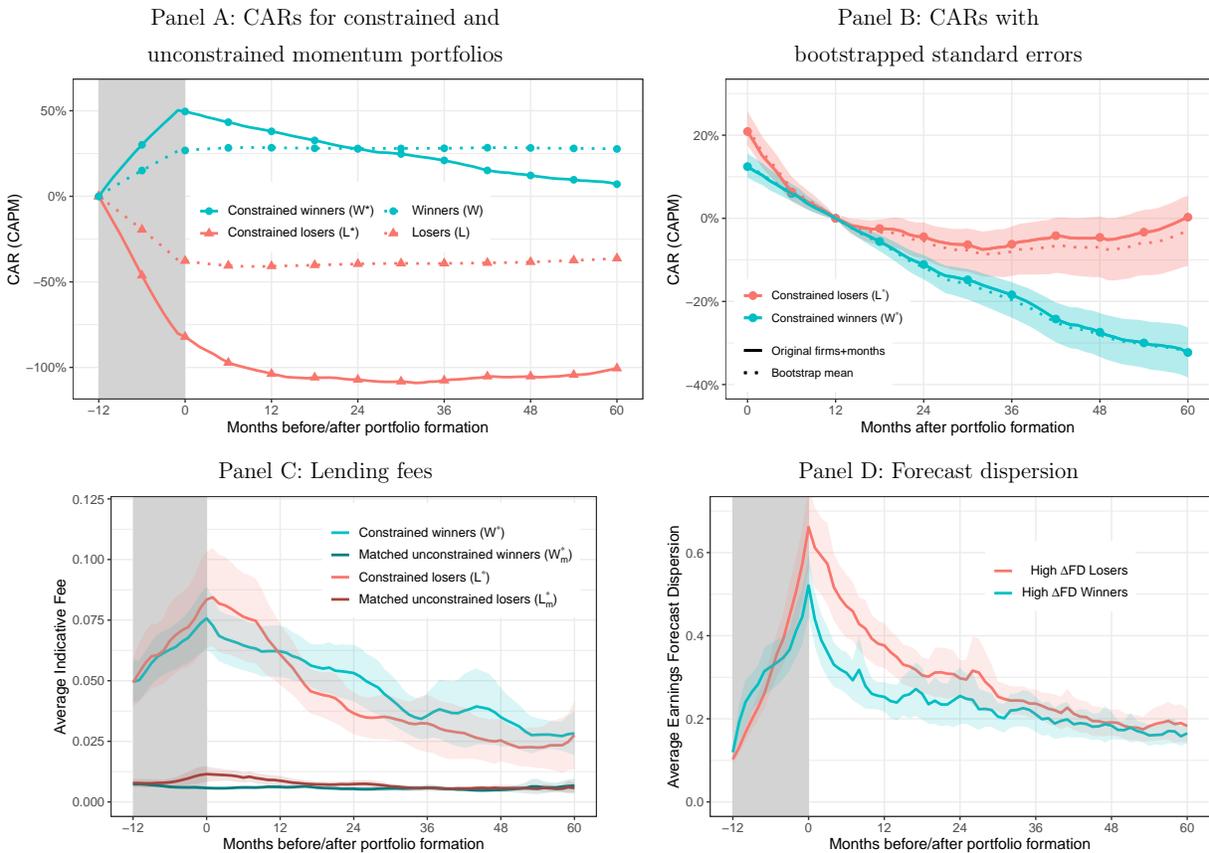
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# Figures

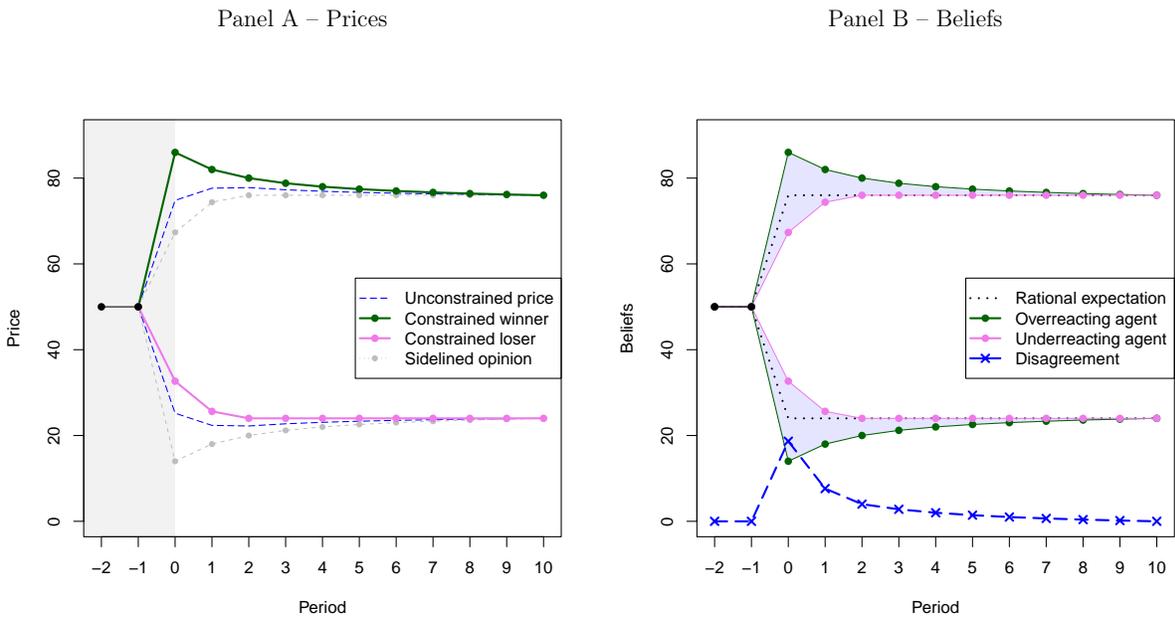
**Figure 1: Returns, fees, and disagreement following positive and negative price shocks**

We calculate abnormal returns for each portfolio for each holding month  $k$  by regressing the time-series of month- $k$  excess returns on the CAPM-MktRF factor. Returns are then cumulated and plotted for the past winner and past loser portfolios (dashed lines) in Panel A. The universe is all US common stocks listed on the NYSE, AMEX, or NASDAQ in the sample from January 1927 to June 2020. Winners are defined as the firms whose returns from 12 months to 1 month before the portfolio-formation date were in the top 30% of all firms, and the past losers are the firms in the bottom 30%. The universe for the solid lines consists of *short-sale constrained* stocks, meaning they are in the bottom 30% of institutional ownership and the top 30% of short interest. For the constrained losers, we additionally impose the condition that they have not been in the constrained-winner portfolio within the past five years, to isolate the long-run effects of winners and losers (see Section 3). The time period for constrained stocks is May 1980 to June 2020. To focus on the differences after 12 months, we center CARs at  $t=12$  in Panel B. We obtain 95% confidence intervals using bootstrapping (details in Appendix B.III). Panel C plots the indicative lending fee from Markit in event time for the constrained winners and losers, as well as portfolios containing matched unconstrained stocks (see Section 3.3). The Markit sample goes from August 2004 to June 2020. Panel D plots earnings forecast dispersion for portfolios independently sorted on past return (30% breakpoints as above) and the one-year change in dispersion of analyst earnings forecasts ( $\Delta FD$ ) from IBES (quintiles, see Section 3.6). In Panels C and D, 95% confidence intervals are based on Newey-West standard errors.



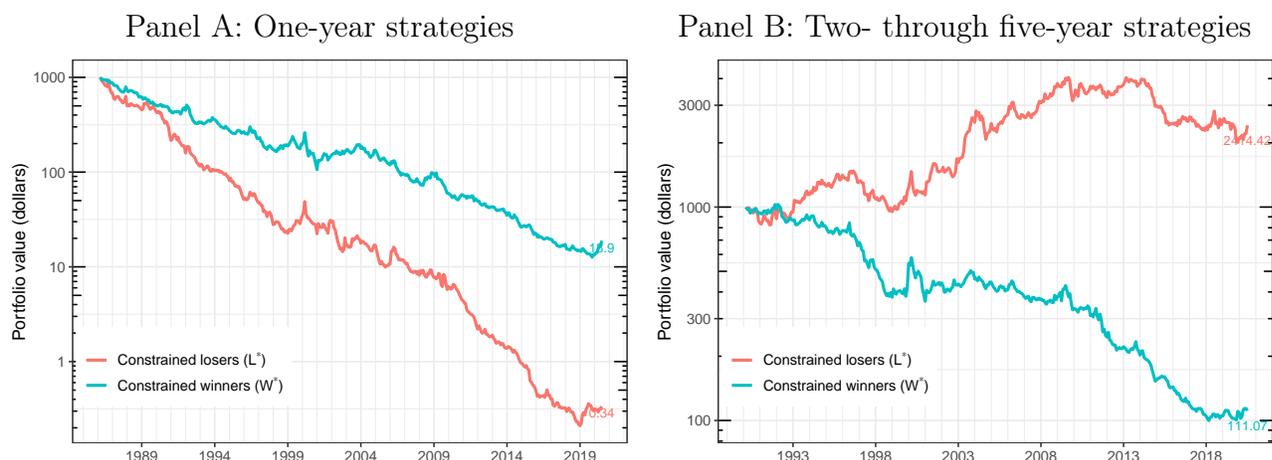
## Figure 2: Dynamics of prices and beliefs

Shown is a stylized representation of the belief dynamics after large price shocks, as implied by the empirical evidence on constrained winners and losers. The green line represents overreacting agents for winners, consistent with the price pattern of constrained winners. Their optimism persists for a relatively long time period (about five years). The pink line represents underreacting agents for losers, as implied by the price pattern of constrained losers. Their optimism persists for a much shorter time period (about a year). The dashed blue lines in Panel A show price paths of unconstrained stocks, assuming the beliefs shown in Panel B.



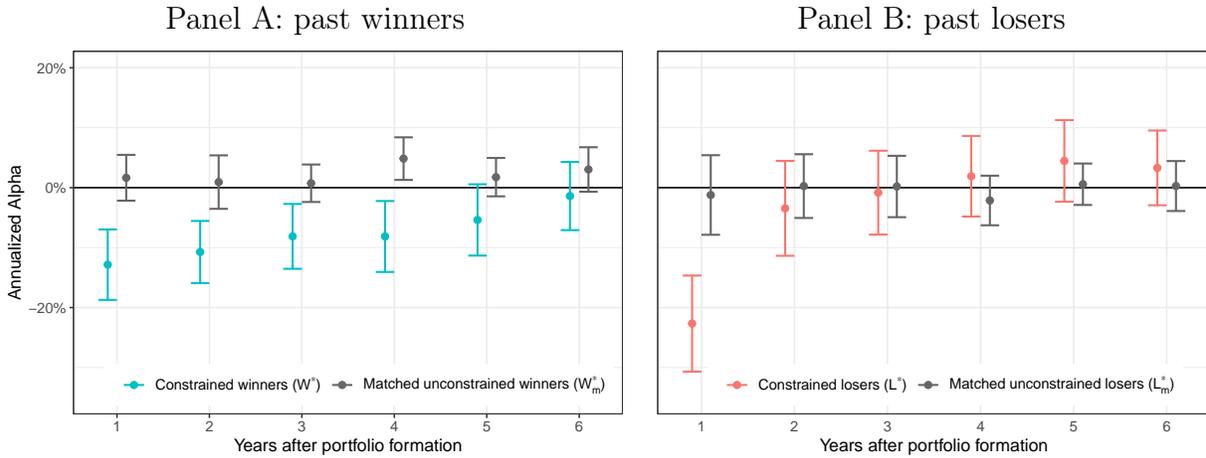
### Figure 3: Performance of hedged constrained strategies over calendar time

This figure presents the investment value for a set of hedged portfolios. To calculate the portfolio value, we assume an investment at the beginning of the sample of \$1,000. We also assume the exposure to the market is hedged. We calculate the hedging coefficient by running a full-sample regression of the strategy excess returns on the market excess returns. Then, using the full-sample regression coefficients, we subtract the returns of the (zero-investment) hedge-portfolio  $[b_{Mkt}(R_{Mkt}-R_{f,t})]$  from the strategy excess returns and add the risk-free rate to generate the hedged strategy returns. Panel A plots the evolution of \$1,000 invested in hedged calendar-time one-year buy-and-hold constrained-winner and -loser (that were not winners in the past 5 years) strategies. Panel B shows results for the corresponding two- to five-year strategies.



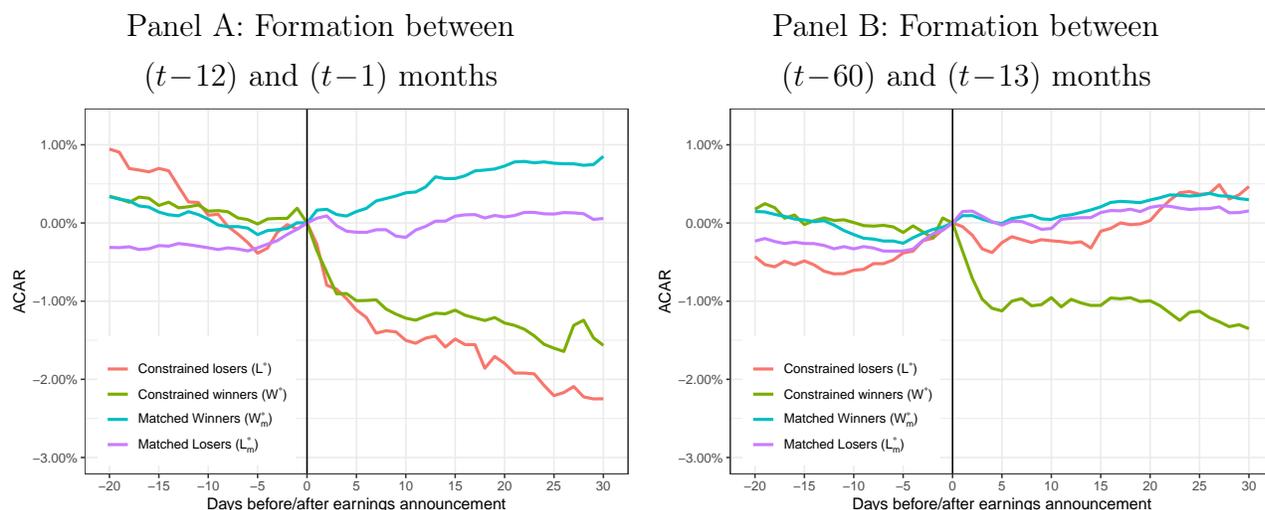
### Figure 4: CAPM-alphas of constrained and matched one-year strategies

The first set of points show the annualized CAPM alphas of strategies of value-weighted portfolios of constrained past winners (Panel A) and losers (Panel B), in years one through six post-formation. The second set of points in Panels A and B are the results of strategies of matched stock-portfolios, based on the Mahalanobis distance calculated on size,  $\log(\text{book-to-market})$ , past return, and short interest. The whiskers represent 95% confidence intervals based on Newey-West standard errors. For details, see [Section 3.3](#).



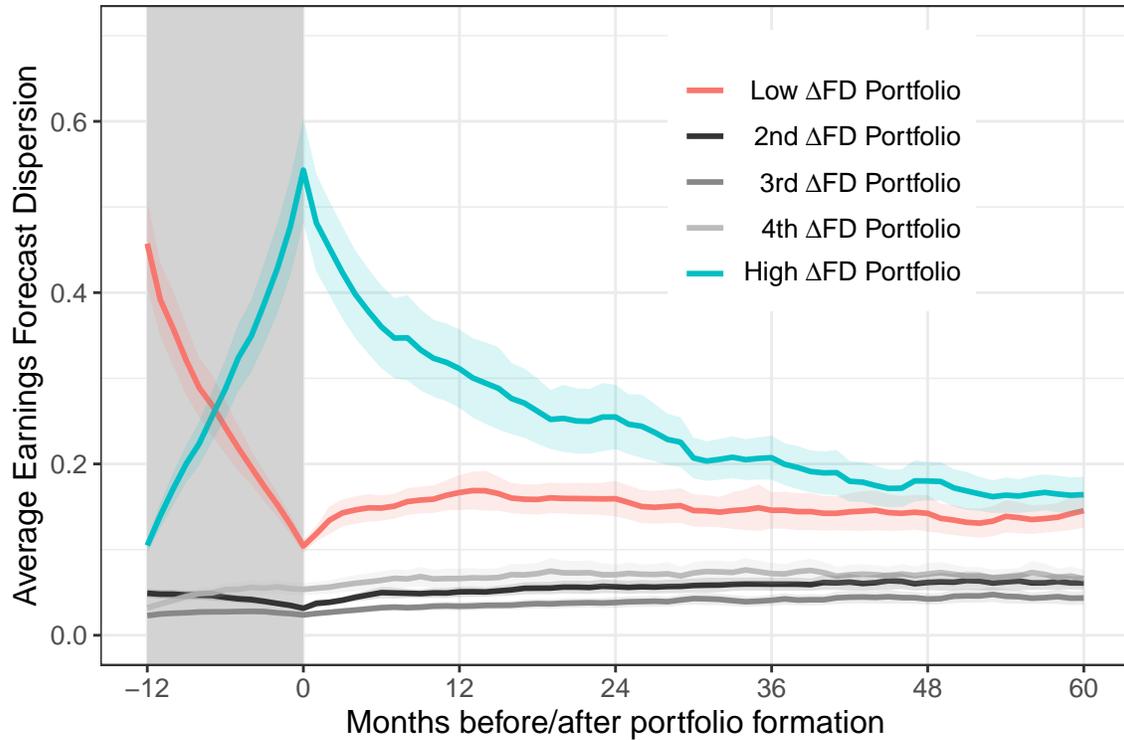
### Figure 5: CAR around earnings announcements

This figure shows cumulated abnormal returns of the constrained winners ( $W^*$ ) and losers ( $L^*$ ) as well as their unconstrained matched counterparts ( $W_m^*$  and  $L_m^*$ ) around the day ( $D=0$ ) of an earnings announcement that occurs in the quarter after portfolio formation (months  $t$  to  $t+2$ ). We include all stocks that were in the respective portfolio in months  $t-12$  through  $t-1$  (Panel A) and  $t-60$  to  $t-13$  (Panel B) and calculate their buy-and-hold weight from formation to each day plotted, by using the price change adjusted by the cumulative price adjustment factor (CFACPR in CRSP). Abnormal returns are calculated by adjusting for beta times the CAPM-market-factor. For each stock, beta is estimated in a one-year window of daily returns prior to the month in which the earnings announcement occurs. To construct the figure, daily abnormal returns are first centered around the day of announcement ( $D=0$ ). They are then cumulated by stock (cumulative abnormal return, CAR) and averaged (ACAR, weighted by the buy-and-hold weight) by portfolio and day relative to announcement. See [Appendix B.V](#) for details.



**Figure 6: Dynamics of earnings forecast dispersion**

Stocks are sorted based on their past one-year change in forecast dispersion into five portfolios. Forecast dispersion is the standard deviation of quarterly earnings forecasts divided by the absolute value of the mean. Data are from IBES from May 1980 to June 2020. The time-series average of the value-weighted portfolio average level of forecast dispersion is tracked over event time, from 12 months before until 60 months after portfolio formation ( $t=0$ ).



## Tables

**Table 1: Characteristics of constrained and matched portfolios**

This table shows time-series averages of value-weighted mean characteristics of the portfolios in the month of portfolio formation. Shown are the average number of stocks, the average market equity (in billion US dollars), return from month t-12 to the end of month t-2 (in %), level of short interest two weeks prior to formation (in %), and change from 11.5 months ago to two weeks ago (in PP), institutional ownership (in % of number of shares outstanding) and its change over the preceding year (in PP), the ratio of book equity of the most recently observed fiscal year to last month's market equity, the average standard deviation of daily idiosyncratic returns in each portfolio (daily, in %) over the month prior to formation (Ang, Hodrick, Xing, and Zhang, 2006), levels (in %) and changes (in PP) over the preceding 12 months in monthly turnover, the level (in %, annualized) and change (in PP, over the preceding 12 months) in the Markit indicative as well as simple average loan fee, the ratio of short interest to institutional ownership (SIRIO) as in Drechsler and Drechsler (2016) (in %), and last, the open-interest weighted average of differences in implied volatilities between matched put and call option pairs at month-end (in %), as in Cremers and Weinbaum (2010). The sample period is April 1985 (to account for the five-year lookback period for losers that were not constrained winners before) to June 2020, except for Markit data, which are available from August 2004. For a comparison with the broader universe of stocks, averages for the remaining portfolios are displayed in Table C.7 in Appendix C.V.

|   | $W^*$  | $L^*$  | $W_m^*$ | $L_m^*$ |
|---|--------|--------|---------|---------|
| Number of stocks                            | 49     | 36     | 49      | 36      |
| Average market equity (B\$)                 | 3.00   | 1.47   | 3.11    | 1.22    |
| Formation-period return (%)                 | 82.36  | -47.37 | 71.37   | -36.07  |
| Institutional ownership (IOR, %)            | 16.91  | 17.55  | 76.33   | 74.40   |
| Change in IOR over preceding year (PP)      | 1.18   | -5.06  | 9.12    | 0.95    |
| Short-interest (SIR, %)                     | 6.52   | 6.41   | 5.36    | 6.33    |
| Change in SIR over preceding year (PP)      | 2.28   | 1.05   | 0.69    | 1.07    |
| Book-to-market ratio                        | 0.30   | 0.91   | 0.35    | 0.90    |
| Idiosyncratic volatility (% , daily)        | 3.04   | 3.95   | 2.11    | 2.69    |
| Turnover (%)                                | 32.64  | 28.22  | 24.36   | 23.51   |
| Change in turnover over preceding year (PP) | 15.95  | 1.86   | 5.16    | 1.65    |
| Ind. fee (%)                                | 6.98   | 7.61   | 0.65    | 1.11    |
| Change in ind. fee over preceding year (PP) | 1.69   | 3.07   | -0.24   | 0.38    |
| Simple avg. fee (SAF, %)                    | 5.26   | 6.59   | 0.43    | 1.08    |
| Change in SAF over preceding year (PP)      | 0.67   | 3.16   | -0.22   | 0.39    |
| SIRIO (%)                                   | 100.31 | 73.30  | 6.47    | 7.79    |
| Option volatility spread (%)                | -5.47  | -6.34  | -0.96   | -1.04   |

**Table 2: One-year strategy performance of constrained and matched portfolios**

This table shows average monthly excess returns (in %, Panel A) as well as results from CAPM (Panel B) and Fama-French-Carhart four-factor regressions (Panel C) for monthly excess returns (in %) of one-year buy-and-hold strategies. To calculate the calendar-time buy-and-hold strategy return, each month, the most recent portfolio is added with \$1, and then the investment amount is not rebalanced for the remaining 12 holding months. The stocks in the constrained portfolios (indicated by  $\cdot^*$ ) were in the lowest group of institutional ownership and the highest group of short interest at some point during months  $\{t - 12, \dots, t - 1\}$  before formation. The columns  $L^*$  ( $W^*$ ) represent the intersections of this constrained portfolio with the lowest (highest) 11-month return lagged by one month. Columns containing a minus sign go long the first and short the second portfolio. Portfolios indicated by  $\cdot_m$  contain unconstrained stocks that were matched to the constrained ones based on size, past return, log(book-to-market), and short interest (indicated by  $\cdot_m$ ) using the Mahalanobis distance. For details, see Section 3.3. *DiD* is the difference-in-differences, namely,  $(W^* - W_m^*) - (L^* - L_m^*)$ . Newey and West (1987) *t*-statistics are shown in parentheses. AvgN is the average number of unique stocks in the portfolio. The row labeled SR displays the Sharpe ratios and IR the information ratios. The sample period is May 1980 to June 2020. The first return is calculated in April 1986, that is, the first time we invested 12 times in a row and had the chance to see if a constrained loser had been a constrained winner over the previous five years.

|                                  | $W^*$            | $L^*$            | $W^*-L^*$        | $W_m^*$          | $L_m^*$          | $W_m^*-L_m^*$    | $W^*-W_m^*$      | $L^*-L_m^*$      | <i>DiD</i>     |
|----------------------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|----------------|
| Panel A: Raw excess returns      |                  |                  |                  |                  |                  |                  |                  |                  |                |
| Average                          | -0.18<br>(-0.54) | -0.90<br>(-1.81) | 0.73<br>(2.00)   | 0.94<br>(3.47)   | 0.83<br>(2.04)   | 0.11<br>(0.43)   | -1.12<br>(-4.38) | -1.74<br>(-4.60) | 0.62<br>(1.80) |
| No. of months                    | 411              | 411              | 411              | 411              | 411              | 411              | 411              | 411              | 411            |
| AvgN                             | 188              | 106              |                  | 262              | 203              |                  |                  |                  |                |
| SR                               | -0.0760          | -0.3076          | 0.3528           | 0.5096           | 0.3676           | 0.0772           | -0.7663          | -0.8728          | 0.3160         |
| Panel B: CAPM regressions        |                  |                  |                  |                  |                  |                  |                  |                  |                |
| Intercept                        | -1.07<br>(-4.28) | -1.89<br>(-5.55) | 0.82<br>(2.27)   | 0.14<br>(0.85)   | -0.10<br>(-0.36) | 0.24<br>(0.98)   | -1.21<br>(-4.52) | -1.79<br>(-4.67) | 0.58<br>(1.70) |
| MktRF                            | 1.37<br>(14.94)  | 1.51<br>(11.97)  | -0.14<br>(-1.02) | 1.23<br>(30.36)  | 1.43<br>(12.76)  | -0.20<br>(-1.56) | 0.14<br>(1.57)   | 0.08<br>(0.61)   | 0.06<br>(0.63) |
| $R^2$                            | 0.5783           | 0.4432           | 0.0081           | 0.7393           | 0.6643           | 0.0343           | 0.0145           | 0.0026           | 0.0014         |
| IR                               | -0.7096          | -0.8637          | 0.4000           | 0.1458           | -0.0774          | 0.1739           | -0.8334          | -0.9000          | 0.2970         |
| Panel C: Four-factor regressions |                  |                  |                  |                  |                  |                  |                  |                  |                |
| Intercept                        | -0.95<br>(-4.07) | -1.37<br>(-3.96) | 0.42<br>(1.20)   | 0.09<br>(0.73)   | 0.21<br>(1.29)   | -0.12<br>(-0.65) | -1.04<br>(-3.79) | -1.58<br>(-3.94) | 0.54<br>(1.50) |
| MktRF                            | 1.17<br>(17.18)  | 1.16<br>(16.01)  | 0.01<br>(0.11)   | 1.15<br>(30.60)  | 1.22<br>(39.35)  | -0.07<br>(-1.44) | 0.02<br>(0.28)   | -0.05<br>(-0.71) | 0.08<br>(0.94) |
| HML                              | -0.31<br>(-2.73) | -0.10<br>(-0.64) | -0.21<br>(-1.27) | -0.04<br>(-0.55) | 0.46<br>(4.22)   | -0.50<br>(-4.60) | -0.28<br>(-1.76) | -0.56<br>(-2.62) | 0.29<br>(2.00) |
| SMB                              | 1.00<br>(11.63)  | 1.22<br>(8.09)   | -0.22<br>(-1.41) | 0.68<br>(8.27)   | 0.96<br>(17.40)  | -0.28<br>(-3.15) | 0.32<br>(2.68)   | 0.25<br>(1.57)   | 0.07<br>(0.45) |
| MOM                              | 0.01<br>(0.09)   | -0.63<br>(-5.18) | 0.64<br>(5.04)   | 0.17<br>(4.09)   | -0.47<br>(-4.38) | 0.64<br>(5.85)   | -0.16<br>(-1.51) | -0.16<br>(-0.85) | 0.00<br>(0.02) |
| $R^2$                            | 0.7505           | 0.6365           | 0.1942           | 0.8645           | 0.8844           | 0.5551           | 0.0932           | 0.0801           | 0.0163         |
| IR                               | -0.8177          | -0.7749          | 0.2281           | 0.1321           | 0.2711           | -0.1276          | -0.7470          | -0.8276          | 0.2788         |

**Table 3: Two- to five-year strategy performance for constrained and matched portfolios**

See caption to Table 2. The only difference here is that we hold stocks that were allocated to one of the portfolios at some point during months  $\{t - 60, \dots, t - 13\}$  before formation. The first monthly return is calculated in April 1990, that is, the first time we invested 48 times in a row.

|                                  | $W^*$            | $L^*$            | $W^*-L^*$        | $W_m^*$          | $L_m^*$          | $W_m^*-L_m^*$    | $W^*-W_m^*$      | $L^*-L_m^*$      | $DiD$            |
|----------------------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Panel A: Raw excess returns      |                  |                  |                  |                  |                  |                  |                  |                  |                  |
| Average                          | 0.23<br>(0.66)   | 1.03<br>(2.81)   | -0.80<br>(-3.87) | 1.01<br>(3.69)   | 0.83<br>(3.01)   | 0.18<br>(1.58)   | -0.78<br>(-4.88) | 0.20<br>(1.01)   | -0.98<br>(-6.38) |
| No. of months                    | 363              | 363              | 363              | 363              | 363              | 363              | 363              | 363              | 363              |
| AvgN                             | 417              | 203              |                  | 621              | 461              |                  |                  |                  |                  |
| SR                               | 0.1094           | 0.4849           | -0.6061          | 0.5848           | 0.4628           | 0.2941           | -0.7263          | 0.1694           | -0.7450          |
| Panel B: CAPM regressions        |                  |                  |                  |                  |                  |                  |                  |                  |                  |
| Intercept                        | -0.74<br>(-3.61) | 0.14<br>(0.59)   | -0.87<br>(-4.34) | 0.17<br>(1.30)   | -0.03<br>(-0.20) | 0.20<br>(1.72)   | -0.90<br>(-5.39) | 0.17<br>(0.89)   | -1.07<br>(-5.76) |
| MktRF                            | 1.43<br>(19.51)  | 1.32<br>(17.57)  | 0.11<br>(1.26)   | 1.25<br>(29.01)  | 1.27<br>(30.90)  | -0.02<br>(-0.48) | 0.18<br>(3.03)   | 0.05<br>(0.70)   | 0.14<br>(1.72)   |
| $R^2$                            | 0.7016           | 0.6029           | 0.0115           | 0.8197           | 0.7893           | 0.0021           | 0.0458           | 0.0025           | 0.0166           |
| IR                               | -0.6280          | 0.1039           | -0.6678          | 0.2255           | -0.0373          | 0.3192           | -0.8619          | 0.1427           | -0.8215          |
| Panel C: Four-factor regressions |                  |                  |                  |                  |                  |                  |                  |                  |                  |
| Intercept                        | -0.62<br>(-4.93) | 0.26<br>(1.20)   | -0.88<br>(-4.33) | 0.14<br>(1.53)   | -0.05<br>(-0.52) | 0.19<br>(2.20)   | -0.76<br>(-5.29) | 0.31<br>(1.38)   | -1.07<br>(-5.24) |
| MktRF                            | 1.25<br>(18.03)  | 1.13<br>(22.15)  | 0.13<br>(1.64)   | 1.17<br>(42.36)  | 1.15<br>(35.76)  | 0.01<br>(0.41)   | 0.08<br>(1.21)   | -0.03<br>(-0.45) | 0.11<br>(1.39)   |
| HML                              | -0.28<br>(-5.08) | 0.11<br>(1.56)   | -0.39<br>(-4.99) | -0.17<br>(-3.78) | 0.17<br>(3.66)   | -0.34<br>(-7.28) | -0.11<br>(-1.16) | -0.06<br>(-0.60) | -0.04<br>(-0.59) |
| SMB                              | 0.74<br>(8.87)   | 0.88<br>(12.29)  | -0.14<br>(-1.73) | 0.51<br>(13.06)  | 0.79<br>(18.48)  | -0.28<br>(-6.17) | 0.22<br>(2.59)   | 0.09<br>(1.14)   | 0.14<br>(1.43)   |
| MOM                              | -0.08<br>(-1.29) | -0.14<br>(-2.44) | 0.06<br>(0.64)   | 0.08<br>(3.55)   | 0.04<br>(1.84)   | 0.05<br>(2.11)   | -0.16<br>(-3.04) | -0.18<br>(-2.18) | 0.02<br>(0.16)   |
| $R^2$                            | 0.8190           | 0.7352           | 0.0893           | 0.9159           | 0.9360           | 0.3511           | 0.1183           | 0.0410           | 0.0277           |
| IR                               | -0.6848          | 0.2342           | -0.7008          | 0.2790           | -0.1146          | 0.3871           | -0.7606          | 0.2641           | -0.8277          |

**Table 4: Regressions of changes in lending fees on event-time dummies**

For each value-weighted portfolio, we calculate the change in the value-weighted Market indicative lending fee (in %, annualized) between months  $t$  and  $t - 1$ . We then regress these changes on indicator variables that control for whether an observation falls into the first year post-formation ( $I_{0 < k \leq 12}$ ), the subsequent four years ( $I_{12 < k \leq 60}$ ), or the year after ( $I_{k > 60 \leq 72}$ ). The first column considers  $W^*$ , and the second considers  $L^*$ . In the final column, we pool all observations into one regression, in order to assess if differences between  $W^*$  and  $L^*$  are statistically significant. Here, we interact all three regressors with a dummy variable that is equal to 1 if an observations comes from a  $W^*$  portfolio. The sample period is from August 2004 until June 2020.

|                      | $W^*$            | $L^*$            | $W^* - L^*$      |
|----------------------|------------------|------------------|------------------|
| $I_{0 < k \leq 12}$  | -0.14<br>(-5.13) | -0.20<br>(-4.69) | 0.06<br>(1.20)   |
| $I_{12 < k \leq 60}$ | -0.09<br>(-6.22) | -0.10<br>(-4.58) | 0.01<br>(0.53)   |
| $I_{k > 60 \leq 72}$ | -0.07<br>(-2.17) | 0.02<br>(0.38)   | -0.09<br>(-1.49) |
| $R^2$                | 0.0060           | 0.0037           | 0.0044           |
| N                    | 11,613           | 11,489           | 23,102           |

**Table 5: Fama-MacBeth regressions of future changes on past changes in forecast dispersion**

The change in forecast dispersion ( $\Delta FD$ ) over the following year (columns 1–2), the next four years (columns 3–4), and the full five-year period (columns 5–6) is regressed on positive and negative changes in forecast dispersion over the previous year. We value-weight observations in the cross-sectional regressions by their market capitalization. Following the [Fama and MacBeth \(1973\)](#) procedure, the time-series average of the regression coefficients is presented. Standard errors are calculated following [Newey and West \(1987\)](#). The time-series average of the cross-sectional  $R^2$  is presented below. The sample period is May 1980 to June 2020.

|                            | $\Delta FD_{t-(t+12)}$ | $\Delta FD_{t-(t+12)}$ | $\Delta FD_{(t+12)-(t+60)}$ | $\Delta FD_{(t+12)-(t+60)}$ | $\Delta FD_{t-(t+60)}$ | $\Delta FD_{t-(t+60)}$ |
|----------------------------|------------------------|------------------------|-----------------------------|-----------------------------|------------------------|------------------------|
| Intercept                  | 0.05                   | 0.04                   | 0.03                        | 0.03                        | 0.07                   | 0.07                   |
|                            | (8.14)                 | (8.11)                 | (2.90)                      | (3.08)                      | (6.85)                 | (6.91)                 |
| $\Delta FD_{(t-12)-t}^+$   | -0.87                  | -0.87                  | -0.13                       | -0.13                       | -0.98                  | -0.98                  |
|                            | (-39.66)               | (-39.00)               | (-5.74)                     | (-5.71)                     | (-73.94)               | (-73.94)               |
| $ \Delta FD_{(t-12)-t}^- $ |                        | 0.06                   |                             | -0.05                       |                        | 0.02                   |
|                            |                        | (3.07)                 |                             | (-1.44)                     |                        | (1.32)                 |
| Avg. $R^2$                 | 0.3839                 | 0.3889                 | 0.0083                      | 0.0149                      | 0.3214                 | 0.3237                 |
| No. of months              | 470                    | 470                    | 427                         | 427                         | 427                    | 427                    |
| Avg. no. of stocks         | 2,015                  | 2,015                  | 1,416                       | 1,416                       | 1,459                  | 1,459                  |

The Online-Appendix with supplemental material can be downloaded here:

<https://www.simonrottke.net/research#TheDynamicsofDisagreement>