Discussion of:

What is Missing in Asset-Pricing Factor Models?

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Basic Idea

- Really nice, provocative paper.
- Takes an APT-based approach to pricing the cross-section
  - systematic and asset-specific risk.
  - the SDF correction term for asset-specific risk is:

$$M^a_{t+1} = -\frac{a' V_e^{-1}}{R_{ft}} e_{t+1}$$

- Finds that:

...more than half of the variation in this SDF is explained by an aggregate measure of asset-specific risk that reflects market frictions and behavioral biases.

Pricing Kernel Variance and Squared Sharpe Ratios

$$
\begin{aligned}
\mathbb{E}[mR] &= 1 \\
\mathbb{E}[mR^e] &= 0 \\
\underbrace{cov(m, R^e)}_{=\rho\sigma_m\sigma_{r^e}} &= \underbrace{\mathbb{E}[mR^e]}_{=0} - \underbrace{\mathbb{E}[m]}_{=\frac{1}{R_f}} \mathbb{E}[R^e] \\
\Rightarrow \sigma_m &= -\rho \cdot \frac{1}{R_f} \cdot \left(\frac{\mathbb{E}[R^e]}{\sigma_{R^e}}\right) \\
\Rightarrow \sigma_m^2 &\approx \mathrm{SR}_{max}^2
\end{aligned}
$$

- So, following Hansen and Jagannathan (1997), the pricing kernel variance is proportional to the maximum squared Sharpe Ratio.

Explaining the Cross Section

Timeline:

- Chen, Roll, and Ross (1986) economic factors:
  - Evidence of that there were premia associanted with innovations in macroeconomic variables, but the Sharpe ratios associated with these portfolios were small.
- Connor and Korajczyk (1988) statistical factors using PCA:
  - effective in explaining the covariance structure, but all but the first PC - which looks like the market - did not carry much of a premium.
- Fama and French (1993): characteristic sorted portfolios:
  - “The 3-factor model does a good job in explaining the cross-section of average returns."

Characteristic-Based Factors

- The Fama and French (1993) characteristic-sort procedure has become standard for forming *factor-portfolios*
- That is, find a characteristic that is associated with expected returns, e.g. book-to-market, and create a corresponding factor portfolio by sorting on this characteristic.
  - The resulting factor portfolio goes long high- and short low-characteristic stocks.
- *Examples:* SMB, HML, RMW, CMA; UMD; WML; LIQ; ISU; QMJ, etc.
  - Fama and French (1993, 2015); Carhart (1997); Daniel and Moskowitz (2016); Pástor and Stambaugh (2003); Daniel and Titman (2006); Asness, Frazzini, and Pedersen (2013); Lustig, Roussanov, and Verdelhan (2011)
  - Cochrane (2011) calls this asset pricing's "factor zoo."

Factor-portfolio inefficiency

- PCA ignores information about expected returns that comes from characteristics.
- Characteristic sorts ignore information about the covariance structure that come from PCA/historical return covariances.
- The characteristic-sorted portfolios can be improved by incorporating the information about the (future) covariance structure
  - this information can come from historical, individual-firm covariances, or from other sources.

Basic Setup

To see this, let's consider a standard setting (with no arbitrage).

- For simplicity (and wlog) assume one priced and one unpriced factor:

$$R_{i,t} = \beta_{i,t-1}\left(f_t + \lambda_{t-1}\right) + \beta^u_{i,t-1} f^u_t + \varepsilon_{i,t} \qquad (1)$$

where

- f_t is a priced factor with premium λ_{t-1},
- f^u_t is an *un*priced factor,

and where

- $\mathbb{E}_{t-1}[f_t] = \mathbb{E}_{t-1}[f^u_t] = \mathbb{E}_{t-1}[\varepsilon_{i,t}] = 0$
- $f_t \perp f^u_t$, $f_t \perp \varepsilon_{i,t}$, $f^u_t \perp \varepsilon_{i,t}$, $\varepsilon_{i,t} \perp \varepsilon_{j,t} \;\; \forall i \neq j$.

Characteristic c as a Proxy for Expected Returns

- We do not observe f_t
- However, suppose there exists an observable characteristic $c_{i,t-1}$ that lines up with expected returns:

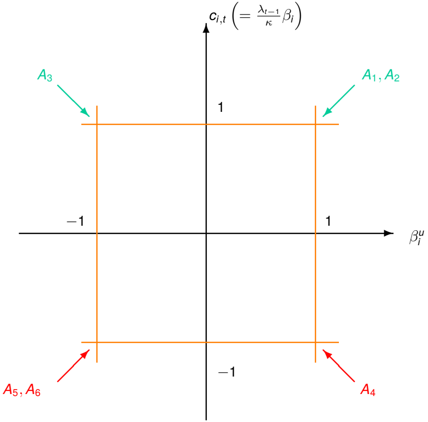
$$\mathbb{E}_{t-1}[R_{i,t}] = \kappa \cdot c_{i,t-1} \tag{2}$$

  - See, e.g., Daniel and Titman (1997) & Fama and French (1993).
- $\Rightarrow$ characteristic perfect proxy for priced factor loading:

$$c_{i,t-1} = \frac{\lambda_{t-1}}{\kappa}\beta_{i,t-1} \tag{3}$$

- Suppose that we form a "factor mimicking portfolio" by buying high c assets and selling low c assets. *Will the resulting portfolio really mimic f_t?*

6 Assets in the Space of Loadings and Characteristics



$$R_t^c = \frac{1}{3} \times [R_{1,t} + R_{2,t} + R_{3,t}] - \frac{1}{3} \times [R_{4,t} + R_{5,t} + R_{6,t}]$$

Characteristics-based Factor R^c is not MVE

- R_t^c is ***not*** mean-variance-efficient
  - It loads on both the priced (f_t) and unpriced (f_t^u) factors.
  - $\Rightarrow$ cannot be the projection of the stochastic discount factor on the space of returns
- How can we improve R_t^c?
  - Construct a hedge portfolio $\boldsymbol{h_t}$ that
    - is strongly correlated with R_t^c $\implies$ large β_h^u; low σ_ϵ
    - has zero expected return $\implies$ $\beta_h = 0$
  - Combine R_t^c and h_t to get
    - same expected return
    - lower volatility

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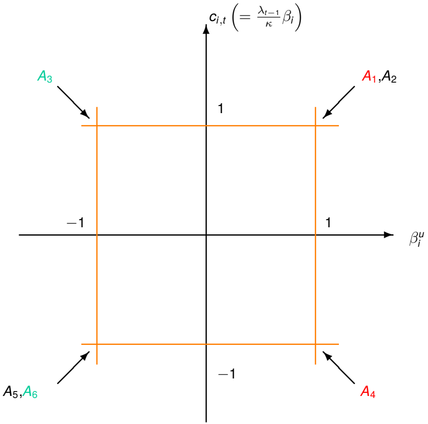


$$h_t = \frac{1}{2}\left[R_{3,t} - R_{1,t}\right] + \frac{1}{2}\left[R_{6,t} - R_{4,t}\right]$$

Improved Factor-Portfolio R^*

- For optimal hedge, project char-based factor portfolio onto hedge portfolio:

$$\begin{aligned} R_t^c &= \gamma h_t + R_t^* \\ \Rightarrow R_t^* &= R_t^c - \gamma h_t \end{aligned}$$

- Optimal hedge ratio:

$$\min_{\gamma} \mathrm{var}\left(R_t^*\right) \quad \Rightarrow \quad \widehat{\gamma} = \frac{\mathrm{cov}\left(R_t^c, h_t\right)}{\mathrm{var}\left(h_t\right)} = \rho_{c,h} \frac{\sigma\left(R_t^c\right)}{\sigma\left(h_t\right)}$$

- Sharpe ratio improvement:

$$\frac{\mathrm{SR}^*}{\mathrm{SR}^c} = \frac{1}{\sqrt{1 - \rho_{c,h}^2}} > 1$$

Generalization

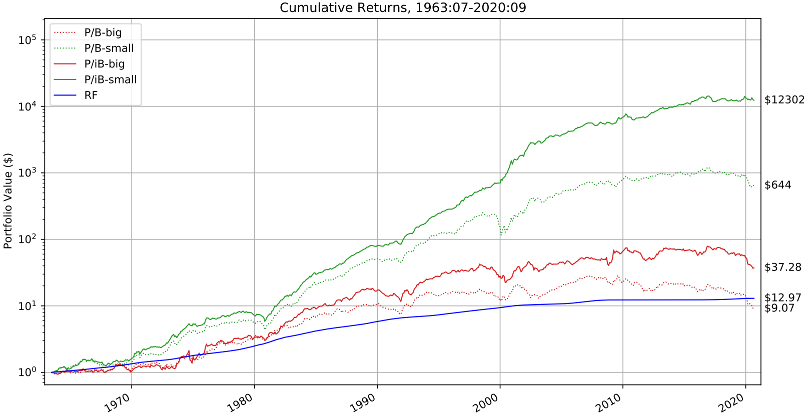
- Our point goes through in a large economy with multiple factors
- The key requirement is that the characteristic c is a good proxy for the expected return.

What is the asset-specific here?

- It is important to note that the asset specific risk that DUZZ document is *not* inividual firm risk.
    - They are clear about this; their 202 basis assets are well-diversified portfolios
- However, they clearly capture components of the returns that are not spanned innovations in economic variables or by the asset-pricing factors proposed in the literature.
    - I would be curious to see the extent to which it is correlated with the hedging portfolio returns.
    - It would also be interesting to see the extent to which it can be captured by better-designed factors.
        - e.g., with size interactions.

Effect of Market Cap

- Given the set of basis assets used, it is perhaps not surprising that the asset-specific-corrected kernel performs so well against size-sorted portfolios.
  - The standard characteristic-based factors do not perform well against small-cap portfolios.



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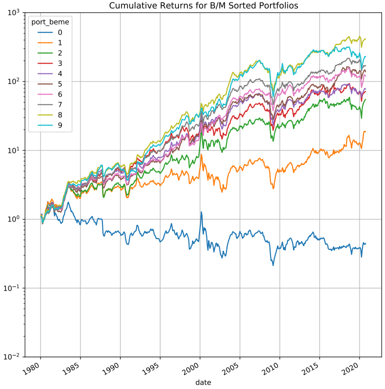
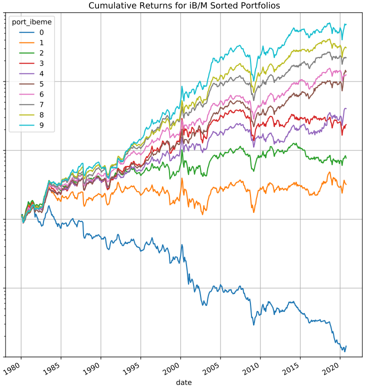
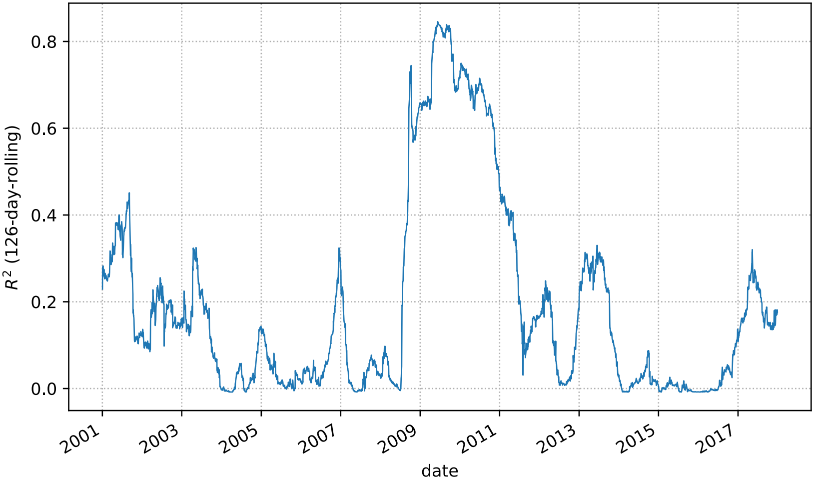




Factor Loadings are not constant for these basis assets

HML & the finance industry in the financial crisis



R^2 of 126-day rolling regressions of HML on finance ("Money") industry return.

Additional Questions

- I'm still a little confused about the finding that "95% of ... the variation [of the systematic component of the SDF] is explained by the market factor"
  - The argument is that "it plays an important role in determining the level of stock returns."
- However, we know that the annualized SR^2 of the market is about 0.16, and the optimal portfolio of even the five-FF factors (Fama and French, 2015) is 1.16.
  - This suggests that a pricing-kernel that explains even the FF-5 factors must have a $\sigma_m^2 \geq 1.16$
    - Note that Daniel, Mota, Rottke, and Santos (2020) show that hedged-versions of these factors have a SR^2 of about 2.16.
    - This is roughly consistent with the finding here that only 44% of the variation in the admissible SDF comes from the systematic component.

Conclusions

- Again, a really nice paper with interesting and provocative results.
- I really like the basic approach of documenting the “asset-specific risk” in the cross section
- I would love to see the authors go further in documenting exactly what the asset-specific component of the pricing kernel really is.

References I

Asness, Clifford S, Andrea Frazzini, and Lasse H Pedersen, 2013, Quality minus junk, AQR Capital Management working paper.

Carhart, Mark M., 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57–82.

Chen, Nai-Fu, Richard Roll, and Stephen A. Ross, 1986, Economic forces and the stock market, *Journal of Business* 59, 383–403.

Cochrane, John H, 2011, Presidential address: Discount rates, *The Journal of Finance* 66, 1047–1108.

Connor, Gregory, and Robert A. Korajczyk, 1988, Risk and return in an equilibrium APT: Application of a new test methodology, *Journal of Financial Economics* 21, 255–289.

Daniel, Kent, Lira Mota, Simon Rottke, and Tano Santos, 2020, The cross section of risk and return, *The Review of Financial Studies* 33, 1927–1979.

Daniel, Kent D., and Tobias J. Moskowitz, 2016, Momentum crashes, *Journal of Financial Economics* 122, 221–247.

Daniel, Kent D., and Sheridan Titman, 1997, Evidence on the characteristics of cross-sectional variation in common stock returns, *Journal of Finance* 52, 1–33.

Daniel, Kent D., and Sheridan Titman, 2006, Market reactions to tangible and intangible information, *Journal of Finance* 61, 1605–1643.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Hansen, Lars P., and Ravi Jagannathan, 1997, Assessing specification errors in stochastic discount factor models, *Journal of Finance* 52, 557–590.

Lustig, Hanno N., Nikolai L. Roussanov, and Adrien Verdelhan, 2011, Common risk factors in currency markets, *Review of Financial Studies* 24, 3731–3777.

Pástor, Ľuboš, and Robert F. Stambaugh, 2003, Liquidity risk and expected stock returns, *Journal of Political Economy* 111, 642–685.