Discussion of:
Asset Prices with Fading Memory

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Introduction

- The paper has two distinct parts:
  - a simple model with some empirical work
  - a more sophisticated Bayesian learning model.
- I'm going to concentrate on the first part.
- In both models agents learn the underlying cashflow growth rate by observing realized cashflow growth rates.
  - As a result of experiential learning, investors overreact to recent growth rates.
  - Agents get no other information.
- I'm going to argue that you need other shocks to explain market returns.

Basic Model - Cash flow process

- The cashflow C_t from the endowment (market) follows GBM with constant drift μ:

$$\Delta c_t = \mu + \epsilon_t$$

where $c_t = \log(C_t)$ and $\epsilon_t \sim iid \;\; \mathcal{N}(0, \sigma_c^2)$

- However, based on Malmendier and Nagel (2016), the *average* agent's belief about μ, $\tilde{\mu}$, follows:

$$\tilde{\mu}_{t+1} = \tilde{\mu}_t + \nu\left(\Delta c_{t+1} - \tilde{\mu}_t\right)$$

  - MN (2016) estimate $\nu = 0.018$/quarter for inflation data.

- implying that:

$$\tilde{\mu}_t = \sum_{j=0}^{\infty} \underbrace{\nu(1-\nu)^j}_{w_j} \Delta c_{t-j}$$

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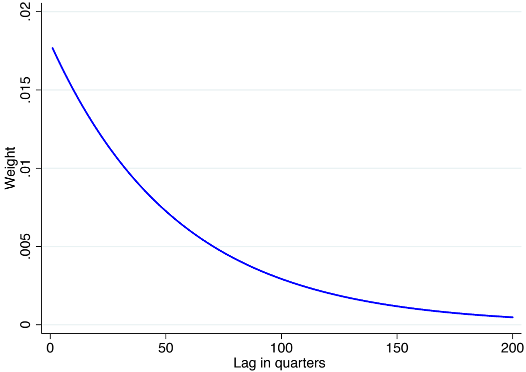
Weighting Function, $\nu = 0.018$

- That is, $\tilde{\mu}_t = \sum_{j=0}^{\infty} w_j \Delta c_{t-j}$, where w_j looks like:



- half life is $\frac{\log(0.5)}{\log(1-\nu)} = 38.2$ quarters ($\sim$ 10 years)
    - $\nu \to 0 \Rightarrow$ "rationality" (*i.e.*, no "fading")

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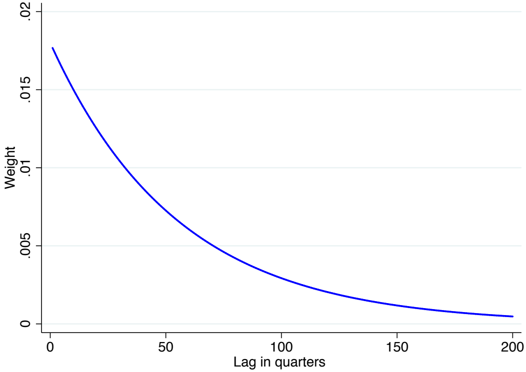

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Basic Model - Pricing

- Representative agent in model sets price to equal PV of future CFs, using constant discount rate of

$$\tilde{\mathbb{E}}_t r_{t+1} = \theta + r_f$$

  - However, agent (mistakenly) extrapolates recent cashflow growth to infer μ.

- Using a Campbell and Shiller (1988) log-linearization:

$$r_{t+1} - \tilde{\mathbb{E}}_t r_{t+1} = \left(1 + \left(\frac{\rho}{1-\rho}\right)\nu\right)(\Delta c_{t+1} - \tilde{\mu}_t)$$

and

$$\mathbb{E}_t r_{t+1} - \underbrace{\tilde{\mathbb{E}}_t r_{t+1}}_{r_f+\theta} = \left(1 + \left(\frac{\rho}{1-\rho}\right)\nu\right)(\mu - \tilde{\mu}_t)$$

implying a negative relationship between recent cashflow growth and future abnormal returns.

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Estimating $\tilde{\mu}_t$

- The authors don't use cashflows to estimate $\tilde{\mu}_t$.
- They instead use historical returns on the market. Effectively, $\tilde{\mu}_{r,t} = \sum_{j=0}^{\infty} w_j r_{t-j}$
- Reasons:
  1. To start in 1926, would need consumption going back to 1876.
  2. "...dividends are influenced by shifts in payout policy that can distort estimates of $\tilde{\mu}$ constructed from dividend growth rates."
  3. The authors simulate $\tilde{\mu}$ and $\tilde{\mu}_r$ (under the null) and show that they are highly correlated.
- A concern is that price shocks will reflect *all* information prices/discount rates.
  - How can we confirm the information that is causing $\mathbb{E}[r]$s to change is cashflow innovations?

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DP decomposition

- I'll show a set of regressions. Data is from Shiller, over the 1946-2014 sample.
- The dependent variable is always the annual real returns on the S&P 500 (R_{t+1})
- The forecasting variables I'll use are:
  1. *dp*: log of preceding year's dividend (D_t), scaled by this year's price (P_t)
  2. *dpL*: dp, lagged 10 years.
  3. Δd: change in the log dividend over the last 10 years.
  4. Δp: change in the log price over the last 10 years.
  5. S: Baker and Wurgler (2000) equity share

An Information Decomposition

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      R   R-squared:                       0.066
Model:                            OLS   Adj. R-squared:                  0.052
No. Observations:                  67   AIC:                            -51.62
Df Residuals:                      65   BIC:                            -47.21
Df Model:                           1
Covariance Type:                  HAC
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.4165      0.157      2.657      0.008       0.109       0.724
dp             0.0983      0.046      2.128      0.033       0.008       0.189
==============================================================================
```

An Information Decomposition

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      R   R-squared:                       0.026
Model:                            OLS   Adj. R-squared:                  0.011
Method:                 Least Squares   F-statistic:                     2.210
Date:                Thu, 10 May 2018   Prob (F-statistic):              0.142
Time:                        09:57:50   Log-Likelihood:                 26.393
No. Observations:                  67   AIC:                            -48.79
Df Residuals:                      65   BIC:                            -44.38
Df Model:                           1
Covariance Type:                  HAC
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0588      0.022      2.676      0.007       0.016       0.102
Delta-d        0.1254      0.084      1.487      0.137      -0.040       0.291
```

- the point estimate on the Δd coefficient is positive, not negative.
    - However, it is statistically insignificant.

dp decomposition

- Consider the identity (like that in Daniel and Titman (2006)):

$$dp_t = dp_{t-10} + \Delta d_{t-10,t} - \Delta p_{t-10,t}$$

- In words, if the market has a high *dp* today, there are three possibilities:
  1. It was high *dp* 10 years ago.
  2. Δd was positive.
  3. Δp was negative.
- At least post-WWII, *dp* forecasts the market.
  - *Which of the three components forecasts the market?*

An Information Decomposition

OLS Regression Results

Dep. Variable:	R	R-squared:	0.111
Model:	OLS	Adj. R-squared:	0.069
Method:	Least Squares	F-statistic:	2.882
Date:	Thu, 10 May 2018	Prob (F-statistic):	0.0427
Time:	09:57:50	Log-Likelihood:	29.465
No. Observations:	67	AIC:	-50.93
Df Residuals:	63	BIC:	-42.11
Df Model:	3		
Covariance Type:	HAC		

	coef	std err	z	P>|z|	[0.025	0.975]
const	0.4635	0.165	2.814	0.005	0.141	0.786
dpL	0.1192	0.051	2.353	0.019	0.020	0.219
Delta-d	0.2698	0.117	2.300	0.021	0.040	0.500
Delta-p	-0.1077	0.051	-2.123	0.034	-0.207	-0.008

- Note that the coefficient on Δd is again positive, and now statistically significant.
  - Suggests that Δd is not just “noise” w.r.t returns.

dp decomposition

- We can also break the market "return" into the part explained by cashflow changes, and the component that isn't (ϵ).

$$\Delta p_{t-10,t} = a \cdot dp_{t-10} + b \cdot \Delta d_{t-10,t} + \epsilon_{t-10,t}$$

- $\epsilon_{t-10,t}$ is the price change over the last 10 years that can't be explained by the growth rate of dividends.
  - The regression $R^2_{adj} = 51.4\%$
  - $t(b = 0) = 6.7$.

An Information Decomposition

```
                              OLS Regression Results
==============================================================================
Dep. Variable:                Delta-p   R-squared:                       0.529
Model:                            OLS   Adj. R-squared:                  0.514
Method:                 Least Squares   F-statistic:                     30.51
Date:                Thu, 10 May 2018   Prob (F-statistic):           4.94e-10
Time:                        10:45:20   Log-Likelihood:                -29.072
No. Observations:                  67   AIC:                             64.14
Df Residuals:                      64   BIC:                             70.76
Df Model:                           2
Covariance Type:                  HAC
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.7185      0.357      4.808      0.000       1.018       2.419
dpL            0.4961      0.109      4.537      0.000       0.282       0.710
Delta-d        1.4854      0.221      6.734      0.000       1.053       1.918
==============================================================================
```

- $R^2_{adj.} = 51.4\% \Rightarrow \rho \approx 0.7,$

An Information Decomposition

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Model:                            OLS   Adj. R-squared:                  0.069
Method:                 Least Squares   F-statistic:                     2.882
Date:                Thu, 10 May 2018   Prob (F-statistic):             0.0427
Time:                        11:21:31   Log-Likelihood:                 29.465
No. Observations:                  67   AIC:                            -50.93
Df Residuals:                      63   BIC:                            -42.11
Df Model:                           3
Covariance Type:                  HAC
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.2784      0.137      2.030      0.042       0.010       0.547
dpL            0.0658      0.043      1.544      0.123      -0.018       0.149
Delta-d        0.1098      0.083      1.328      0.184      -0.052       0.272
resid         -0.1077      0.051     -2.123      0.034      -0.207      -0.008
==============================================================================
```

- The coefficient on *resid* is exactly the same as in the previous regression.
- The coefficients on dp_{t-10} and Δd are what they would be were resid not included in the regression.

An Information Decomposition

OLS Regression Results

Dep. Variable:	R	R-squared:	0.166
Model:	OLS	Adj. R-squared:	0.104
Method:	Least Squares	F-statistic:	2.878
Date:	Thu, 10 May 2018	Prob (F-statistic):	0.0311
Time:	09:57:51	Log-Likelihood:	28.634
No. Observations:	59	AIC:	-47.27
Df Residuals:	54	BIC:	-36.88
Df Model:	4		
Covariance Type:	HAC		

	coef	std err	z	P>|z|	[0.025	0.975]
const	0.6982	0.227	3.077	0.002	0.253	1.143
dpL	0.1585	0.068	2.318	0.020	0.024	0.293
Delta-d	0.3394	0.124	2.728	0.006	0.096	0.583
Delta-p	-0.1589	0.057	-2.765	0.006	-0.272	-0.046
S	-0.5252	0.283	-1.858	0.063	-1.079	0.029

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Campbell, John Y., and Robert J. Shiller, 1988, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–228.

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