Discussion of:
The Mystery of Currency Betas

by:
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The carry trade has long been a puzzle in asset pricing. Let’s look at the data from Lustig, Roussanov, and Verdelhan (2011), who sort 35 currencies into six portfolios (P1-P6) based on currency interest rates relative to the dollar interest rate.
LRV Portfolios – Cumulative Returns

- P1 final value is $0.60; P6 final value is $5.84
- $t(\bar{R}_{HML_{FX}} = 0) = 4.77; SR_{HML_{FX}} = 0.87$ (annualized)
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LVR Factors

Lustig, Roussanov, and Verdelhan (2011) also find that two return-based risk factors “explain” the cross-section of currency returns:

- $R_{RX}$: the cross-sectional average return on all six currency portfolios.
- $R_{HML_{FX}}$: the difference in the returns on portfolio 6 and portfolio 1.
### Asset Return Space

#### Panel II: Factor Betas

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>(a_0)</th>
<th>(\beta_{HMLFX}^j)</th>
<th>(\beta_{RX}^j)</th>
<th>(R^2)</th>
<th>(\chi^2(\alpha))</th>
<th>(p)-value</th>
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<tbody>
<tr>
<td>1</td>
<td>-0.10</td>
<td>-0.39</td>
<td>1.05</td>
<td>91.64</td>
<td></td>
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</tr>
<tr>
<td></td>
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<td>[0.02]</td>
<td>[0.03]</td>
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</tr>
<tr>
<td>2</td>
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<tr>
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<td>5</td>
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<td>[0.50]</td>
<td>[0.02]</td>
<td>[0.03]</td>
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</tr>
<tr>
<td>All</td>
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<td></td>
<td>6.79</td>
<td>34.05%</td>
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<table>
<thead>
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<th>(\beta_{HMLFX}^j)</th>
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<tr>
<td></td>
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<tr>
<td></td>
<td>-0.17</td>
<td>0.12</td>
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<td>0.36</td>
<td>0.49</td>
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</table>

All: 2.63 75.64%

**Note that:**

- the \(R^2\)s for these time-series regressions are 75%-94% (81%-94% for developed currencies).
- There is considerable variation in \(\beta_{HMLFX}^j\).
- \(\beta_{RX}^j \approx 1\) for all portfolios.
Economic Factor Models

- In the absence of arbitrage, all excess returns $R_{t+1}$ are priced by a stochastic discount factor (pricing kernel) $\tilde{m}$ such that:

$$E_t[\tilde{m}_{t+1} \tilde{R}_{t+1}] = 0$$

or, equivalently,

$$E_t[\tilde{R}_{t+1}] = -\text{cov}(\tilde{m}_{t+1}, \tilde{R}_{t+1}).$$

where $E_t[\tilde{m}_{t+1}] = 1$.

- For rational investors, and in the absence of frictions, $\tilde{m}_{t+1}$ is the ratio of marginal utilities at time $t+1$ and time $t$.
- This equation is just the FOC for investor optimization.
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- For rational investors, and in the absence of frictions, $\tilde{m}_{t+1}$ is the ratio of marginal utilities at time $t + 1$ and time $t$.
  - This equation is just the FOC for investor optimization.
Thus, it should be possible to link $\tilde{m}$ to a set of factors which proxy for innovations in marginal utility:

$$\tilde{m}_{t+1} = a_t + b_t' \tilde{f}_{t+1}$$

For this reason, a number of macro-finance researchers have proposed macro-based models which deliver a pricing kernel which can “explain” the premia interest-rate sorted portfolios.
Theoretically Motivated Factors

1. Rare Disasters
   - Farhi and Gabaix (2008):

2. Habit-Based Explanation – Consumption/Surplus Ratio
   - Verdelhan (2010)

3. Share of World Consumption (SWC)
   - Colciato and Croce (2013)

In addition, Riddiough examines a set of 6 financial risk factors and 6 separate macroeconomic risk factors:

- Each of the 6 financial risk factors are found to be priced, with the t-statistic over 3.0 for the composite financial risk.
- Each of the 6 macro risk factors have $t > 2.0$; for four factors, $t > 3.0$. 
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This Paper’s Approach

Figure 5: The Distribution of Simulated t-statistics. The figure presents histograms of simulated t-statistics when pricing test asset portfolios using artificially constructed factors. In the top-left corner, the histogram is constructed by simulating 20,000 factors by randomly assigning currencies from the All Countries sample to one of five portfolios each month. The factors are constructed by taking the difference in returns on the fifth and first portfolios. The test asset portfolios are five currency portfolios sorted on the basis of the forward premia. High interest currencies are included in the fifth portfolio, while low interest currencies are included in the first portfolio. The t-statistics are based on the factor price of risk of the simulated factor, estimated using the Fama-MacBeth procedure, when pricing the five forward-premia-sorted portfolios, in addition to the DOL risk factor. In the top-right figure, the 20,000 factors are constructed by creating a dollar-neutral portfolio by randomly allocating weights to each of the five forward-premia-sorted portfolios each month. The factors are re-scaled such that they are always long and short one dollar. In the bottom two figures, while the factors remain the same, more test asset portfolios are included. In the bottom-right figure, both factors (DOL and HML) are included, as well as the five randomly constructed portfolios. In the bottom-right figure, only the two risk factors are added to the five forward-premia-sorted portfolios. Data on forward and spot exchange rates are from Barclays and Reuters, available on DataStream. The sample is from 1983 to 2011.
Economically Motivated Explanations

The finding that there are a number of weakly correlated factors, all of which seem to “explain” the carry trade, is reminiscent of the literature on economic explanations of the value premium (for equities).

I’m going to present some results from Daniel and Titman (2012) that dealt with this topic, with the goal of explaining this puzzling result from a slightly different point of view.
## Economically Motivated Explanations

<table>
<thead>
<tr>
<th>Paper</th>
<th>Factor(s)</th>
<th>Cond. Vars.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conditional (C)CAPM Models</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ferson and Harvey (1999)</td>
<td>VW</td>
<td>S&amp;P 500 Dividend Yield</td>
</tr>
<tr>
<td>Lettau and Ludvigson (2001)</td>
<td>VW or Cons Growth</td>
<td>cay</td>
</tr>
<tr>
<td>Santos and Veronesi (2005)</td>
<td>VW + Labor Income Growth</td>
<td>Labor Income to Cons Ratio (s)</td>
</tr>
<tr>
<td><strong>Alternative-Factor Models</strong></td>
<td></td>
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</tr>
<tr>
<td>Fama and French (1993)</td>
<td>VW, HML, SMB</td>
<td>DEF</td>
</tr>
<tr>
<td>Piazzesi, Schneider, and Tuzel (2007)</td>
<td>Cons Growth + ΔNH Expenditure Ratio (Δlog(α))</td>
<td>Housing Collateral Ratio</td>
</tr>
<tr>
<td>Lustig and Nieuwerburgh (2002)</td>
<td>Scaled Rental Price Change (AΔlogρ)</td>
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<tr>
<td>Aït-Sahalia, Parker, and Yogo (2004)</td>
<td>Luxury Good Consumption</td>
<td></td>
</tr>
<tr>
<td>Li, Vassalou, and Xing (2006)</td>
<td>Sector Inv. Growth Rates</td>
<td></td>
</tr>
<tr>
<td>Parker and Juillard (2003)</td>
<td>Innovations in Future Long Horizon Consumption Growth</td>
<td></td>
</tr>
<tr>
<td>Campbell and Vuolteenaho (2004)</td>
<td>CF and DR news</td>
<td></td>
</tr>
</tbody>
</table>
Given this equivalence, and based on the results of these studies, there are more than a dozen factors that appear to "explain" the value effect.

Interestingly, it turns out that the proposed factors and scaled factors are not highly correlated.
## Sample Correlation Matrix for Candidate Factors

- **Quarterly Data; 1963Q4:1998Q3**

<table>
<thead>
<tr>
<th></th>
<th>HML</th>
<th>DP·$r_m$</th>
<th>$\hat{\text{cay}}·r_m$</th>
<th>$s·r_m$</th>
<th>$\hat{\text{cay}}·\Delta c$</th>
<th>$\Delta y$</th>
<th>$\Delta (\text{prop})$</th>
<th>$\Delta \log(\alpha)$</th>
<th>$N_{CF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HML</td>
<td>1</td>
<td>-0.10</td>
<td>0.07</td>
<td>-0.05</td>
<td>0.06</td>
<td>0.01</td>
<td>0.07</td>
<td>0.11</td>
<td>0.27</td>
</tr>
<tr>
<td>DP·$r_m$</td>
<td>-0.10</td>
<td>1</td>
<td>0.61</td>
<td>0.37</td>
<td>0.14</td>
<td>-0.01</td>
<td>0.04</td>
<td>0.00</td>
<td>-0.09</td>
</tr>
<tr>
<td>$\hat{\text{cay}}·r_m$</td>
<td>0.07</td>
<td>0.61</td>
<td>1</td>
<td>0.03</td>
<td>0.12</td>
<td>-0.03</td>
<td>-0.16</td>
<td>-0.00</td>
<td>-0.12</td>
</tr>
<tr>
<td>$s·r_m$</td>
<td>-0.05</td>
<td>0.37</td>
<td>0.03</td>
<td>1</td>
<td>0.07</td>
<td>0.03</td>
<td>0.14</td>
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<td>0.07</td>
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<tr>
<td>$\hat{\text{cay}}·\Delta c$</td>
<td>0.06</td>
<td>0.14</td>
<td>0.12</td>
<td>0.07</td>
<td>1</td>
<td>0.13</td>
<td>0.10</td>
<td>-0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>0.01</td>
<td>-0.01</td>
<td>-0.03</td>
<td>0.03</td>
<td>0.13</td>
<td>1</td>
<td>0.25</td>
<td>0.15</td>
<td>-0.10</td>
</tr>
<tr>
<td>$\Delta (\text{prop})$</td>
<td>0.07</td>
<td>0.04</td>
<td>-0.16</td>
<td>0.14</td>
<td>0.10</td>
<td>0.25</td>
<td>1</td>
<td>0.28</td>
<td>0.11</td>
</tr>
<tr>
<td>$\Delta \log(\alpha)$</td>
<td>0.11</td>
<td>0.00</td>
<td>-0.00</td>
<td>-0.07</td>
<td>-0.07</td>
<td>0.15</td>
<td>0.28</td>
<td>1</td>
<td>0.09</td>
</tr>
<tr>
<td>$N_{CF}$</td>
<td>0.27</td>
<td>-0.09</td>
<td>-0.12</td>
<td>0.07</td>
<td>0.06</td>
<td>-0.10</td>
<td>0.11</td>
<td>0.09</td>
<td>1</td>
</tr>
</tbody>
</table>
Why do each of these factor explanations seem to “work?”

- That is, they fail to reject the proposed factor model.

The surprising answer is that this is because they are all correct.
Return Space Geometry

- Fama and French (1993) (Table 6) run time-series regressions for each of the 25 SZ/BM sorted portfolios:

$$\tilde{R}_{i,t} - RF_t = a + b \cdot (\tilde{R}_{m,t} - RF_t) + h \cdot \tilde{HML}_t + s \cdot \tilde{SMB}_t + \tilde{\epsilon}_t$$

- The $R^2$s are:

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
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<tr>
<td>Small</td>
<td>0.94</td>
<td>0.96</td>
<td>0.97</td>
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<tr>
<td>2</td>
<td>0.95</td>
<td>0.96</td>
<td>0.95</td>
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</tr>
<tr>
<td>3</td>
<td>0.95</td>
<td>0.94</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>4</td>
<td>0.94</td>
<td>0.93</td>
<td>0.91</td>
<td>0.89</td>
<td>0.89</td>
</tr>
<tr>
<td>Big</td>
<td>0.94</td>
<td>0.92</td>
<td>0.88</td>
<td>0.90</td>
<td>0.83</td>
</tr>
</tbody>
</table>

- In addition, the estimates of $b$ range from 0.91 to 1.18 (std-dev = 0.06).
This means that the returns of these 25 portfolios, net of the market return, lie *approximately* in a 2-dimensional excess return space $\mathbb{R}^{e*}$ spanned by HML and SMB:
In any test where the $\lambda$s are free parameters, a test of a single-factor model with the 25 FF portfolios is a test of whether $\text{corr}(f^*, R_{MVE}^e) = 1$
Moreover, with two factors, assuming \( f_1^* \neq k \cdot f_2^* \), some linear combination of the \( \tilde{f} \)s will always price the assets.

- Any \( f_1^* \) and \( f_2^* \) form a basis for the subspace.
The problem is that any $b'f$ such that

$$b'f = \tilde{R}_{MVE}^\theta + \tilde{\epsilon}, \text{ for } \epsilon \perp \text{HML, SMB}$$

will price the 25 portfolios.

- However, some caveats are:
  - Again, the space is only *approximately* 2-dimensional.
  - Ridiculous factor risk premia ($\lambda$s) may be required.

Thus, to increase the power of the test, the test asset space must be augmented in the direction of $\epsilon$. 
Test Power

- Any test of an asset pricing model is a test of whether the vector of pricing errors of a set of portfolios ($\alpha$) is zero.
- The usual way to test this set of moment restrictions is to form the test statistic:
  \[ \hat{\alpha}' \Omega \hat{\alpha}. \]
  which is asymptotically central $\chi^2$ distributed under the null hypothesis.
- Under the alternative hypothesis, it is non-central $\chi^2$ with NCP $\alpha'_A \Omega \alpha_A$. 
References I


References II


Lustig, Hanno, and Stijn Van Nieuwerburgh, 2002, Housing collateral, consumption insurance and risk premia, University of Chicago working paper.

