Discussion of:

Consumption Risk and Cross-Sectional Returns

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Outline

- Consumption Based Asset Pricing in Frictionless, RE Models
- Explaining Market & Risk-Free Asset Returns
  - The equity premium & correlation puzzles.
- Explaining Size and Book-to-Market Sorted Portfolios
  - Sharpe Ratios and Correlations.
- How Long Horizons Could Potentially Help
  - consumption, return serial correlations & cross-correlations
  - Hansen-Jagannathan analysis
- What might explain the discrepancies?

Frictionless RE Model Implications:

- The pricing equation in discrete time is:

$$P_{i,t} = E_t \left[\tilde{m}_{t+1} \, \tilde{Y}_{i,t+1} \right]$$

  - for any asset, portfolio, or dynamic trading strategy i

Frictionless RE Model Implications:

- The pricing equation in discrete time is:

$$1 = E_t \left[\tilde{m}_{t+1} \, \tilde{R}_{i,t+1} \right]$$

  - in Gross-Return Form

Frictionless RE Model Implications:

- The pricing equation in discrete time is:

$$0 = E_t\left[\tilde{m}_{t+1}\,\tilde{r}_{i,t+1}\right]$$

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- This is just the FOC from the maximization problem, and hence is valid for *any* investor, for *any* asset or portfolio of assets,

Frictionless RE Model Implications:

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$$0 = E_t\left[\tilde{m}_{t+\tau}\,\tilde{r}_{i,t+\tau}\right]$$

  - for *any* excess-return $r_{i,t+\tau}$, for any τ
- This is just the FOC from the maximization problem, and hence is valid for *any* investor, for *any* asset or portfolio of assets,
- *and over any time period*
  - *e.g.*, from t to $t+\tau$

Model Implications – HJ Bounds

- From the covariance definition:

$$cov(\tilde{m}, \tilde{r}_i) = E[\tilde{m}\,\tilde{r}_i] - E[\tilde{m}]E[\tilde{r}_i]$$

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  - using $E[\tilde{m}\,\tilde{r}_i] = 0$, gives:

$$cov(\tilde{m}, \tilde{r}_i) = -E[\tilde{m}]E[\tilde{r}_i]$$

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- use $cov(m, r) = \sigma_m \sigma_r \rho_{m,r}$ to get:
$$\frac{\sigma_m}{E[\tilde{m}]} = \left(\frac{-1}{\rho_{m,r}}\right)\left(\frac{E[\tilde{r}_i]}{\sigma_r}\right)$$

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- Finally, using $\rho_{m,r} > -1$ gives the Hansen and Jagannathan (1991) bound:
$$\frac{\sigma_m}{E[\tilde{m}]} \geq \frac{E[\tilde{r}_i]}{\sigma_r}$$

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- We know from the previous literature (see, *e.g.*, Cochrane and Hansen (1992)) that the Market Sharpe ratio is high relative to consumption volatility, so we need a high CRRA to explain just the market risk premium:

Implications for Risk-Aversion

- For example, if the representative agent has:

$$U(C_t) = \beta^t \frac{C^{1-\gamma}}{1-\gamma}$$

then

$$\tilde{m}_{t+\tau} = \frac{U'(\tilde{C}_{t+\tau})}{U'(C_t)} = \beta^\tau \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$$

- Then, taking logs:

$$log(\tilde{m}_{t+\tau}) = \underbrace{\tau log(\beta)}_{\text{assume}=0} - \gamma \Delta c_{t+\tau}$$

  - where $\Delta c_{t+\tau}$ is the change in $log(C)$ from t to $t+\tau$.

Implications for Risk-Aversion (2)

- then, use $e^x = 1 + x + \frac{x^2}{2} + \ldots$, to get

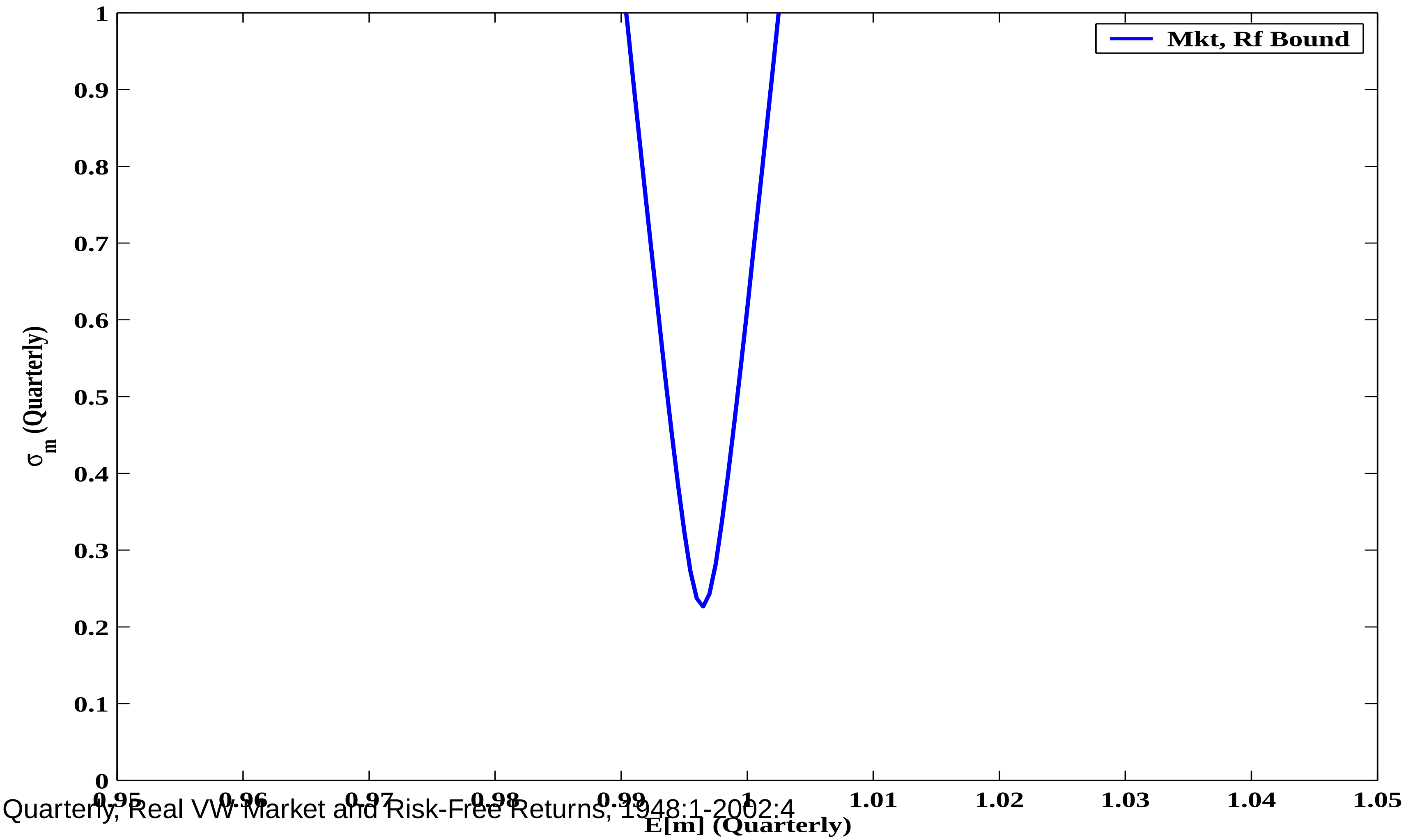
$$\tilde{m}_{t+\tau} = 1 - \gamma \Delta c_{t+\tau} \left(+ \frac{\gamma^2}{2} (\Delta c)^2 + \cdots \right)$$

- This means that $\sigma_m \approx \gamma \sigma_c$ and

$$\gamma \frac{\sigma_c}{E[\tilde{m}]} \geq \frac{E[\tilde{r}_i]}{\sigma_r}$$

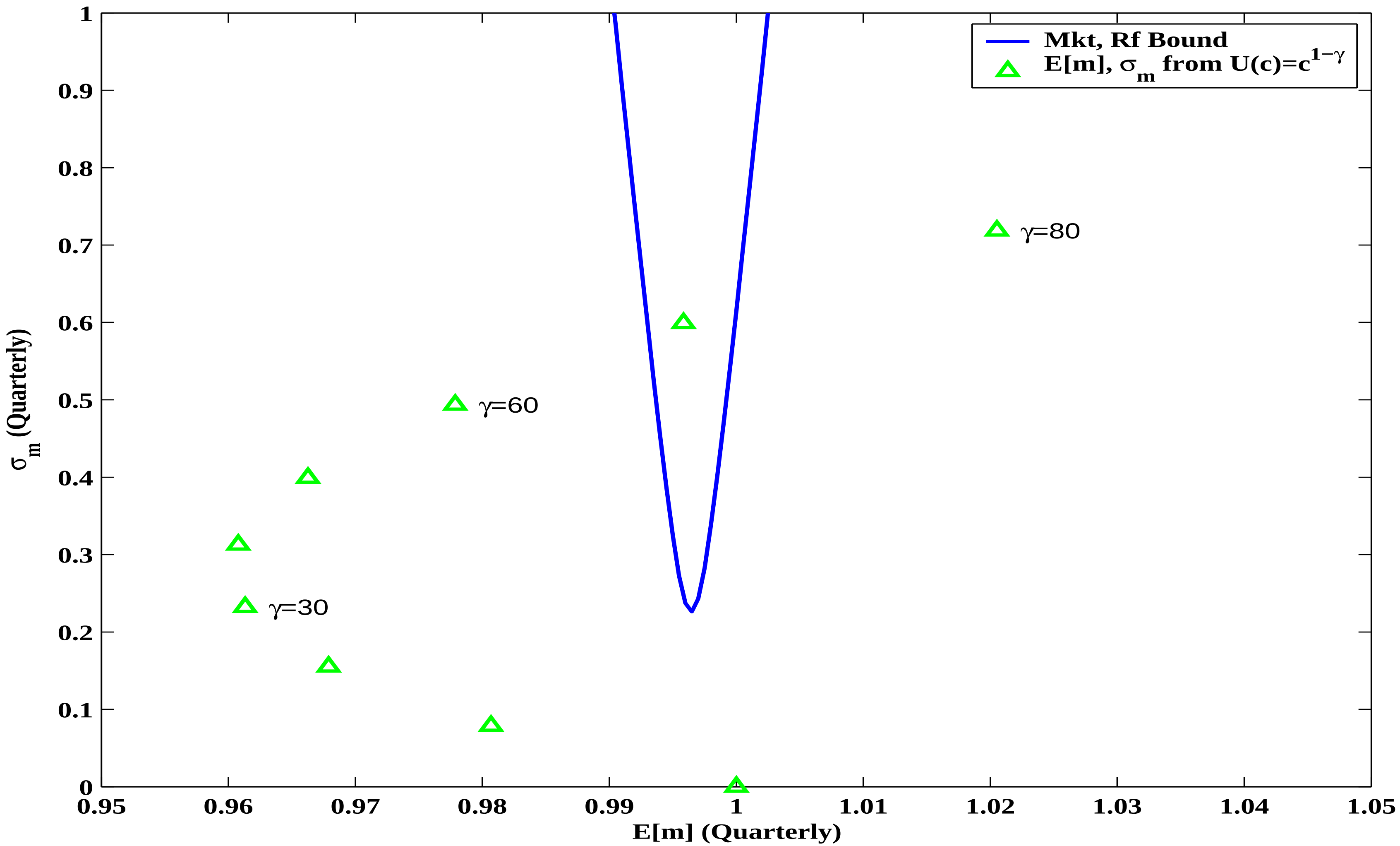
- Since σ_c is small, we need a big γ to explain the high market Sharpe ratio.
  - This is the equity premium puzzle.

Hansen-Jagannathan Bounds



Quarterly, Real VW Market and Risk-Free Returns, 1948:1-2002:4

Hansen-Jagannathan Bounds



Quarterly, Real VW Market and Risk-Free Returns and real nondurable per-capita PCEs, 1948:1-2002:4

The Correlation Puzzle

- However, it is actually a bit worse than this:

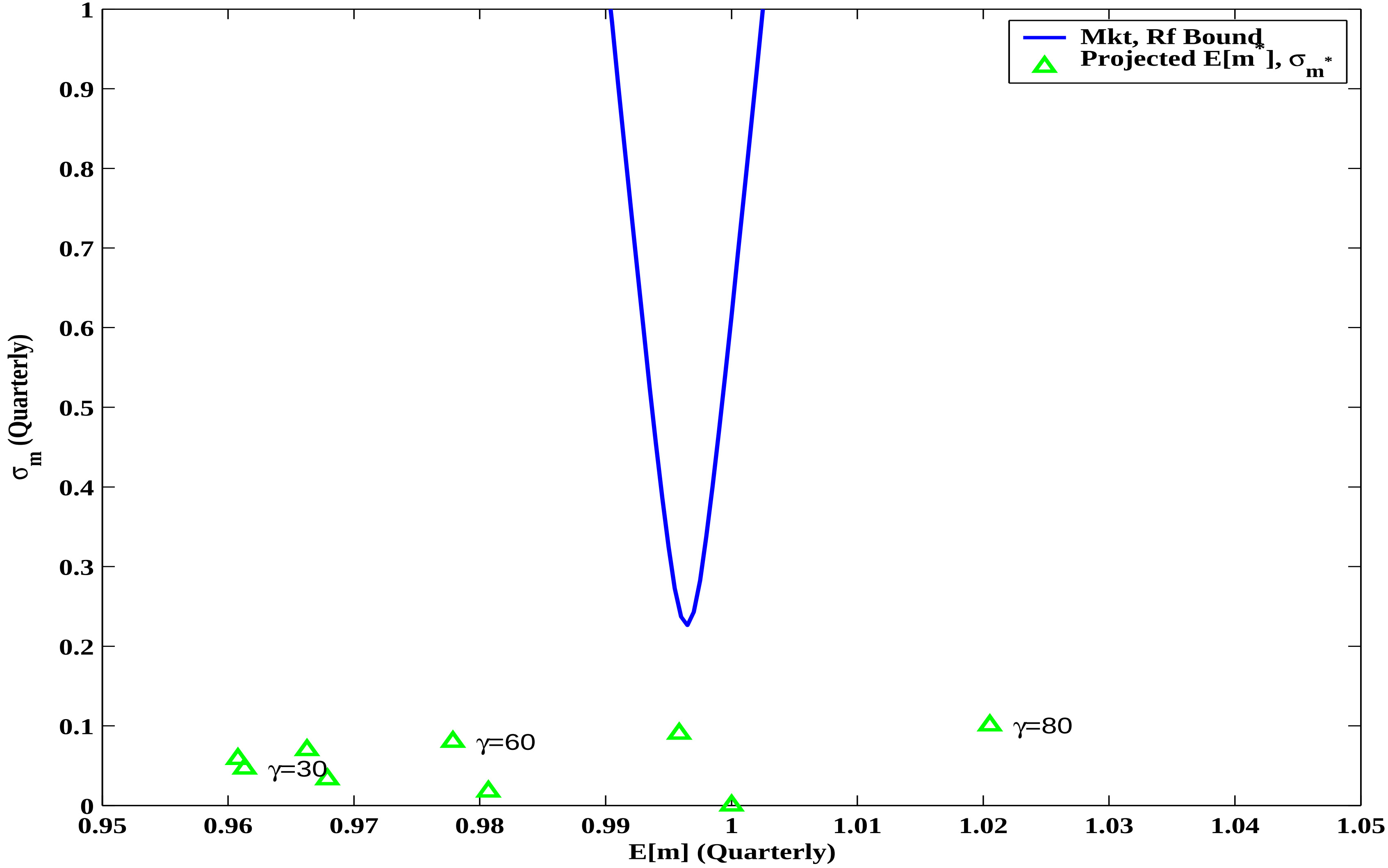
$$\frac{\sigma_m}{E[\tilde{m}]} = \left(\frac{-1}{\rho_{m,r}}\right)\left(\frac{E[\tilde{r}_i]}{\sigma_r}\right)$$

- The problem is that the correlation between consumption growth innovations and the market return is about 10%.
- This suggests that the bound is really a factor of $\sim$10 worse.

The Correlation Puzzle

- Cochrane and Hansen (1992) suggest examining HJ plots which use the calculated m^* rather than m
  - m^* is the projection of m onto the asset return space.
  - If m is a valid pricing kernel, then m^* will also be a vaild pricing kernel.

Hansen-Jagannathan Bounds



Quarterly, Real VW Market and Risk-Free Returns and real nondurable per-capita PCEs, 1948:1-2002:4

Summary – Equity Premium Puzzle

- For the market:

$$\frac{\sigma_m}{E[\tilde{m}]} = \left(\frac{-1}{\rho_{m,r}}\right)\left(\frac{E[\tilde{r}_i]}{\sigma_r}\right)$$

1. σ_m is too small.
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- Now, what happens when we:
  1. include size and BM sorted portfolios?
  2. move to long horizons?

Cross-Section of Average Returns

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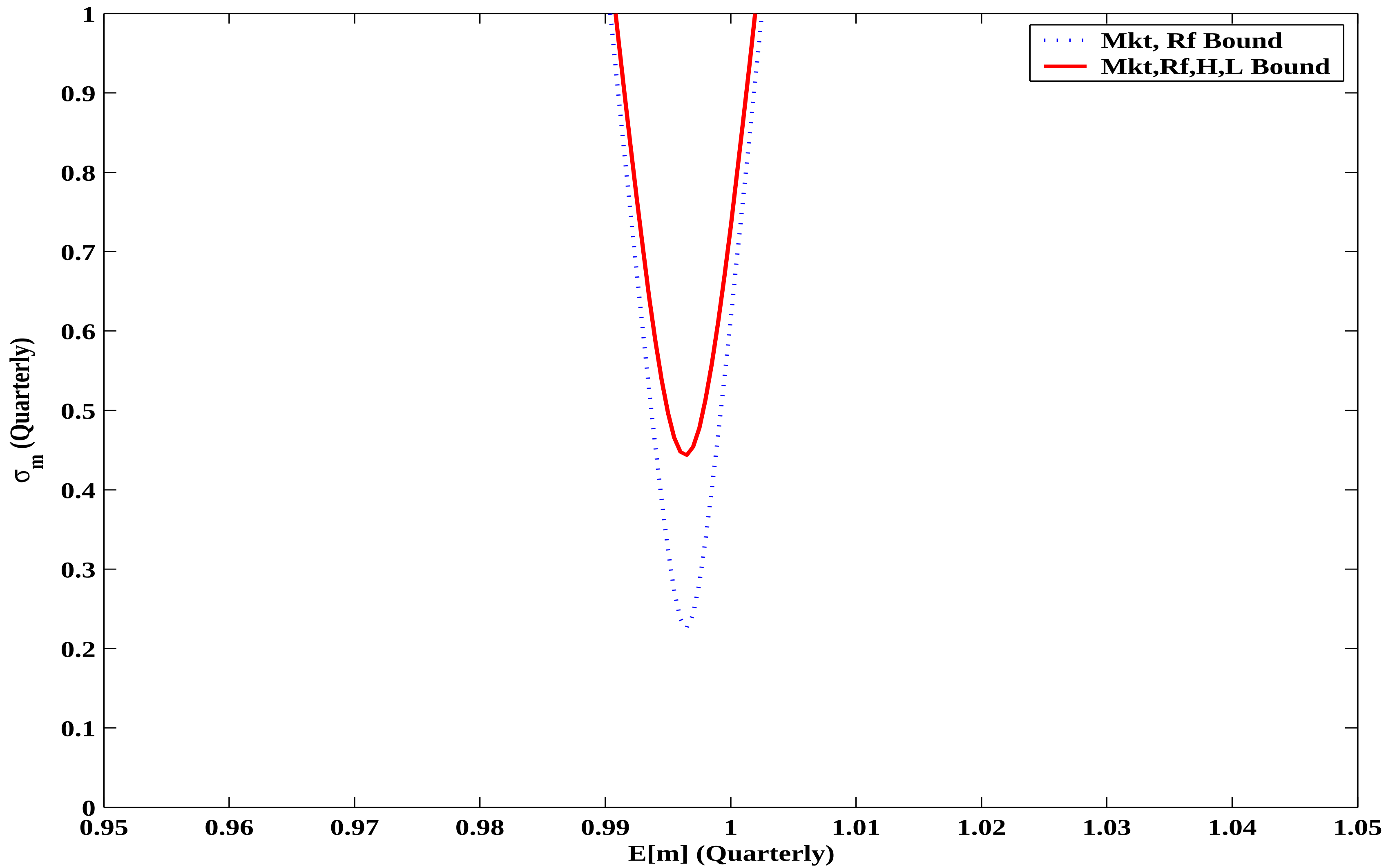
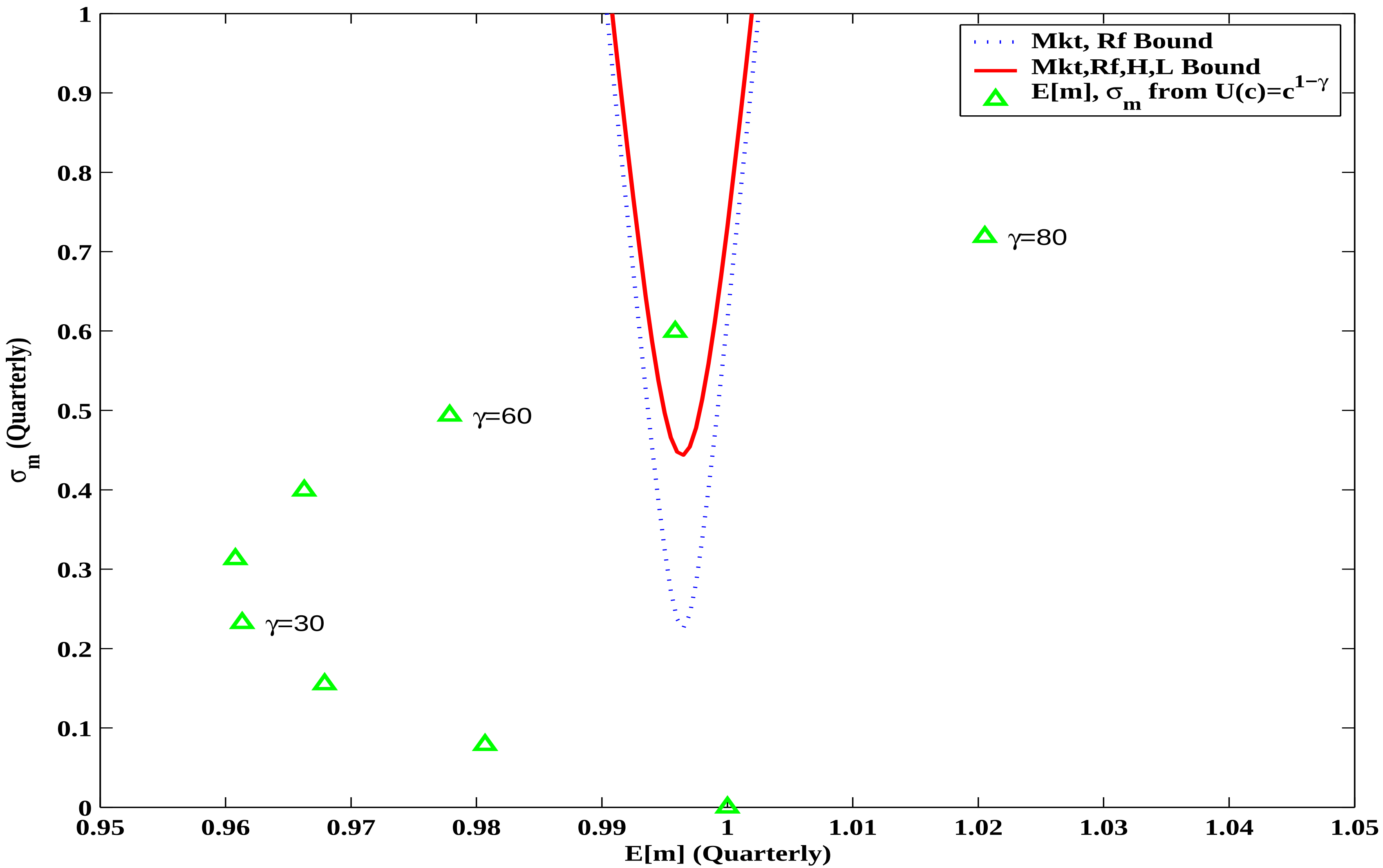
  3. Pastor and Stambaugh (2003): *Liquidity*,
- These anomalies produce apparently large Sharpe Ratios (MacKinlay (1995))
- Additionally, the returns from these strategies are even less correlated with consumption growth than is the market.

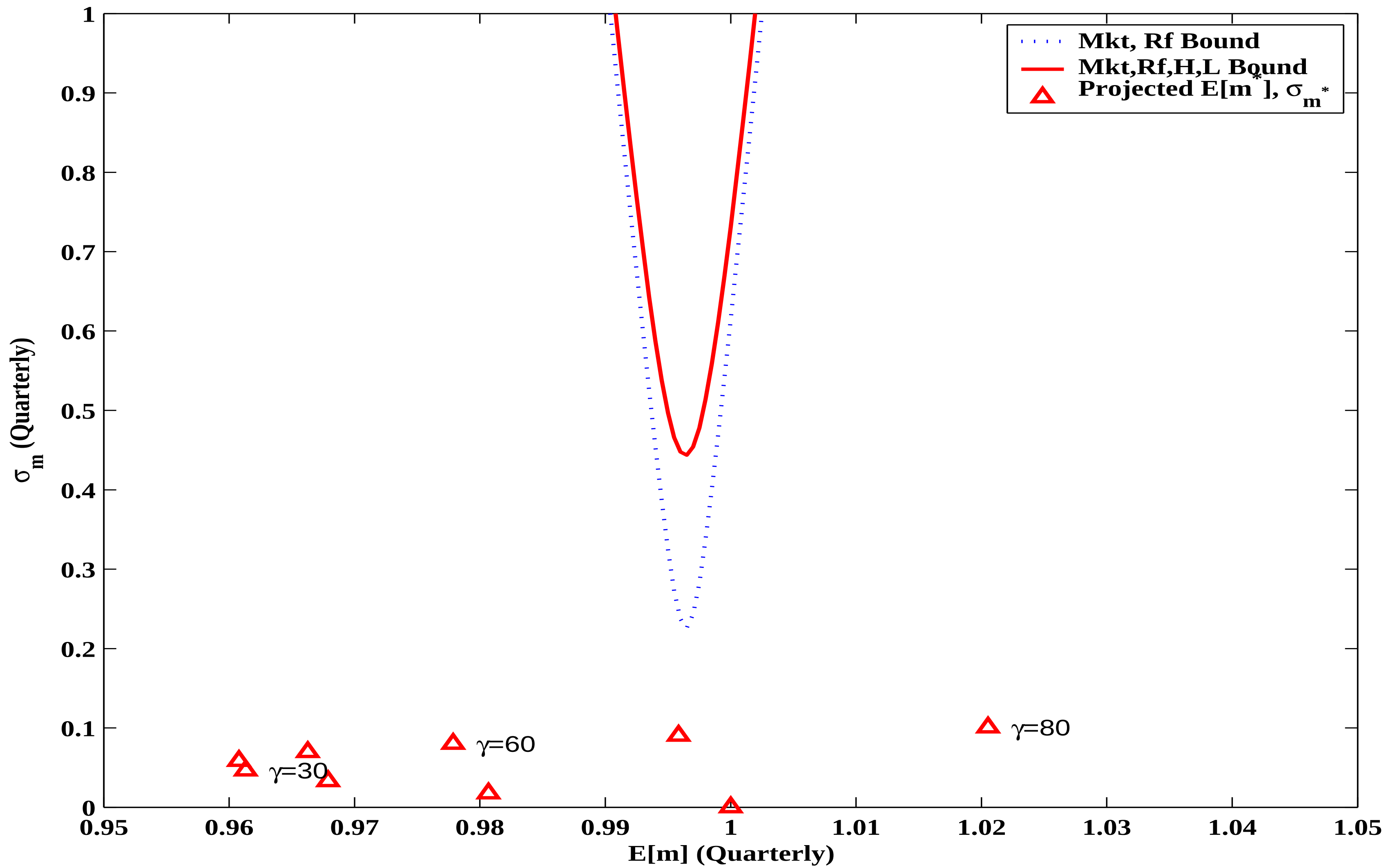
Hansen-Jagannathan Bounds



Hansen-Jagannathan Bounds



Hansen-Jagannathan Bounds



Can Long Horizons Help?

- The same HJ bound restrictions apply at long-horizons:

$$\frac{\sigma_m}{E[\tilde{m}]} \geq \left(\frac{E[\tilde{r}_i]}{\sigma_r} \right)$$

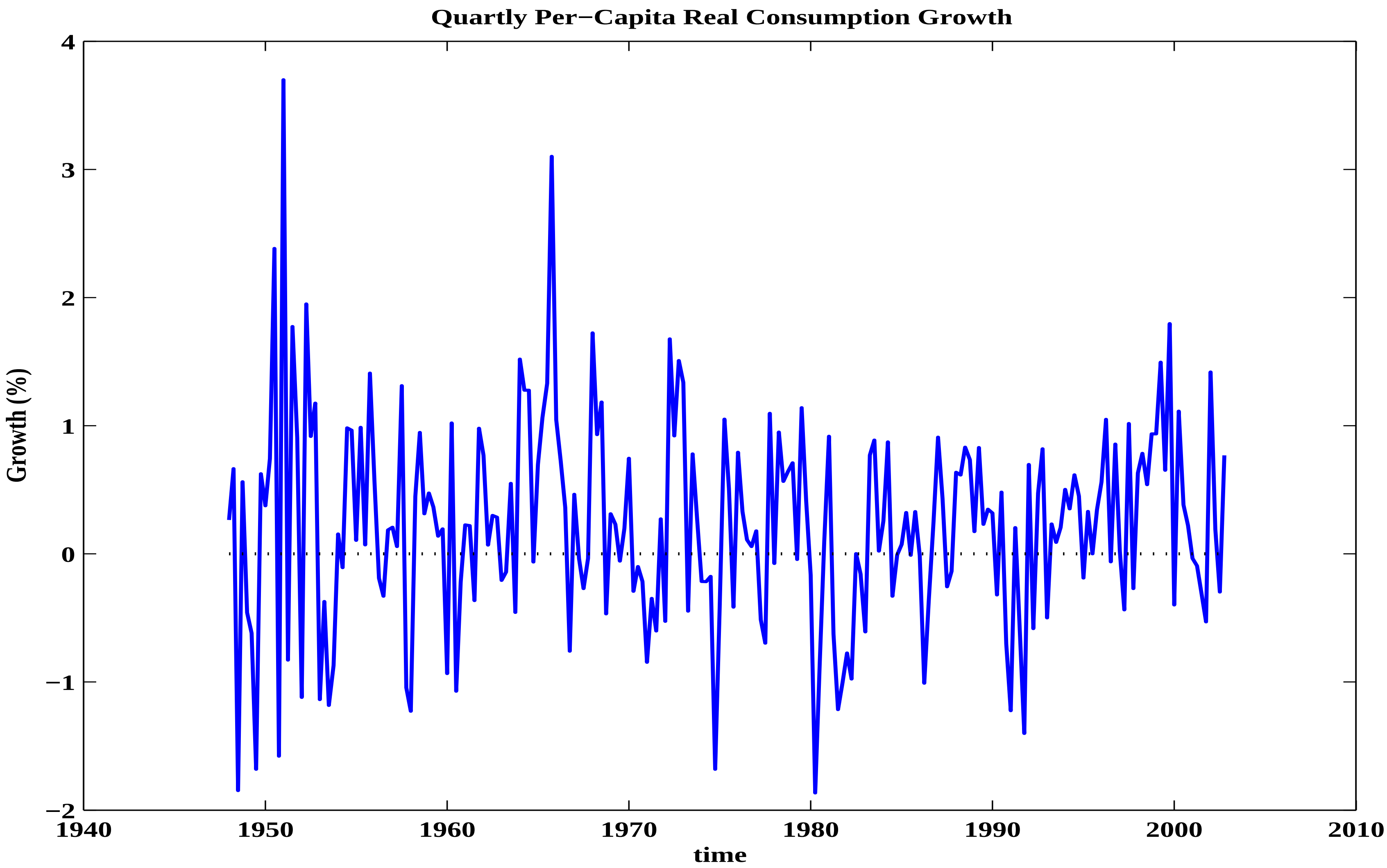
- However, moving to long-horizons won't help if:
  1. marginal utility growth is serially uncorrelated.
  2. returns are serially uncorrelated
  - In this case both sides of the HJ bound will be $\sim \sqrt{\tau}$.

Can Long Horizons Help?

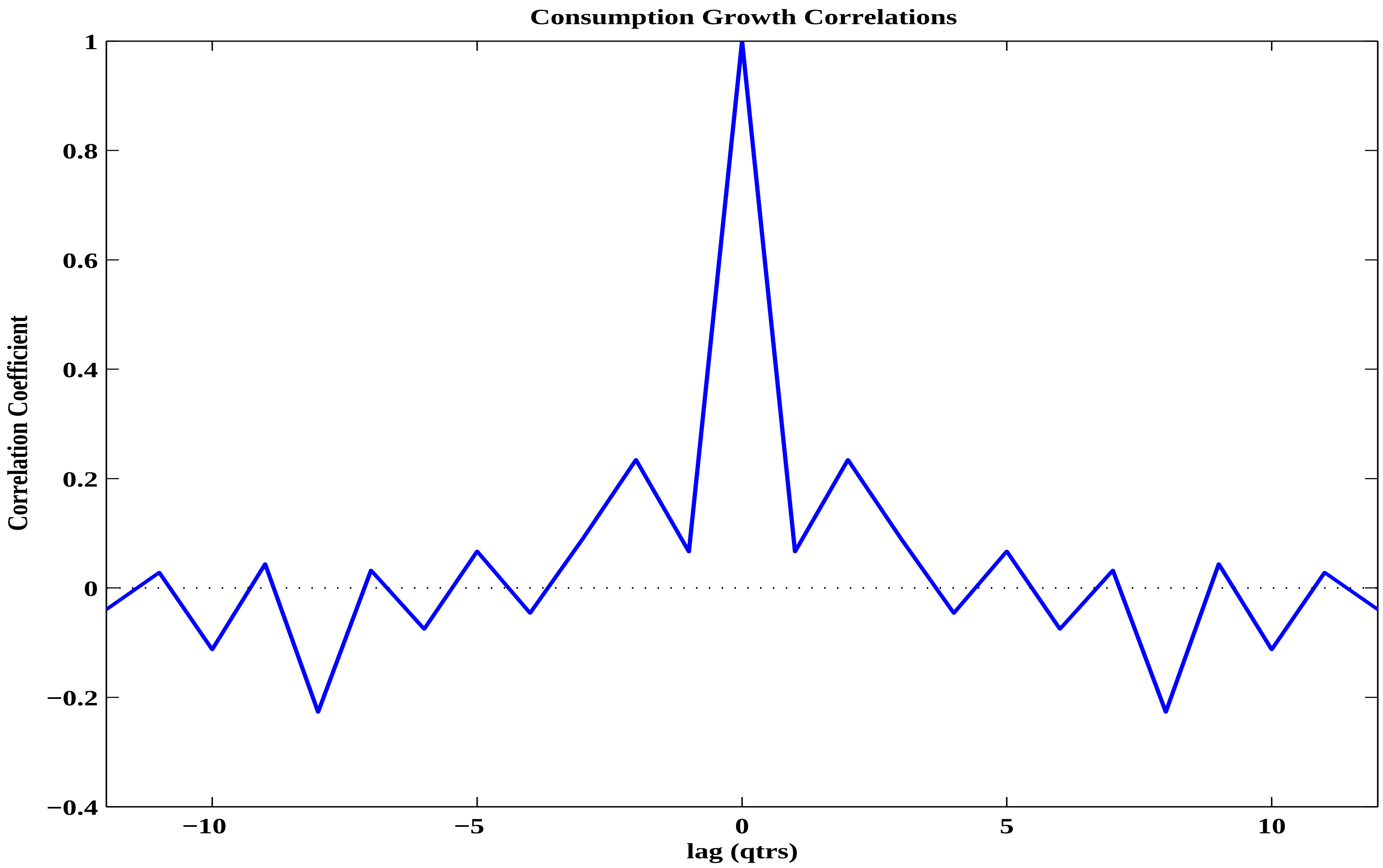
$$\frac{\sigma_m}{E[\tilde{m}]} \geq \left(\frac{E[\tilde{r}_i]}{\sigma_r} \right)$$

- For a long horizons consumption-based model to work (without extreme preferences) it will have to be the case that there is either:
  1. strong positive serial correlation in consumption growth (and *calculated* marginal utility)
  2. strong negative correlation in the portfolio returns*
- Also, the maximum Sharpe ratio portfolio should have a strong negative correlation with the long-horizon pricing kernel.

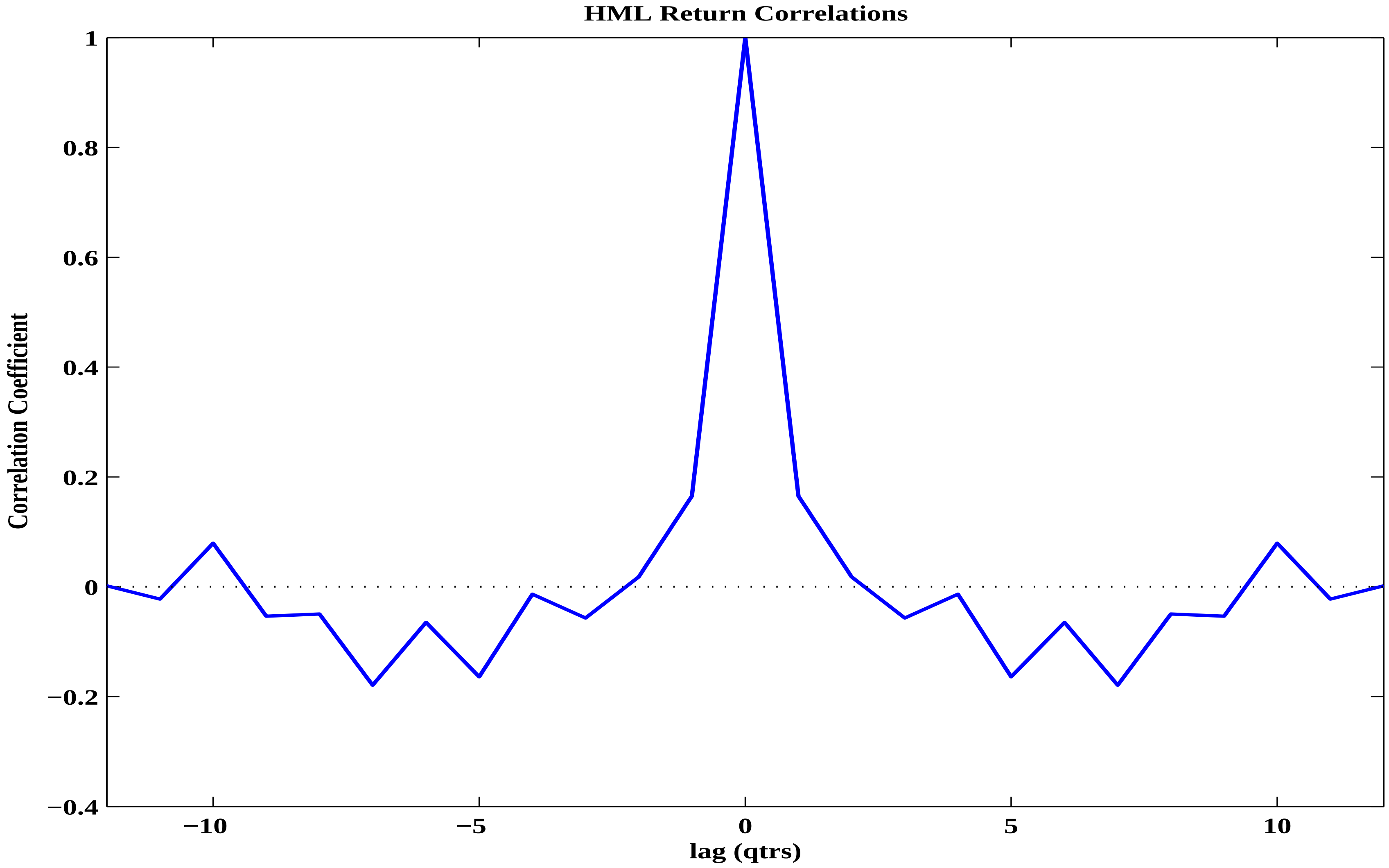
The Consumption Data



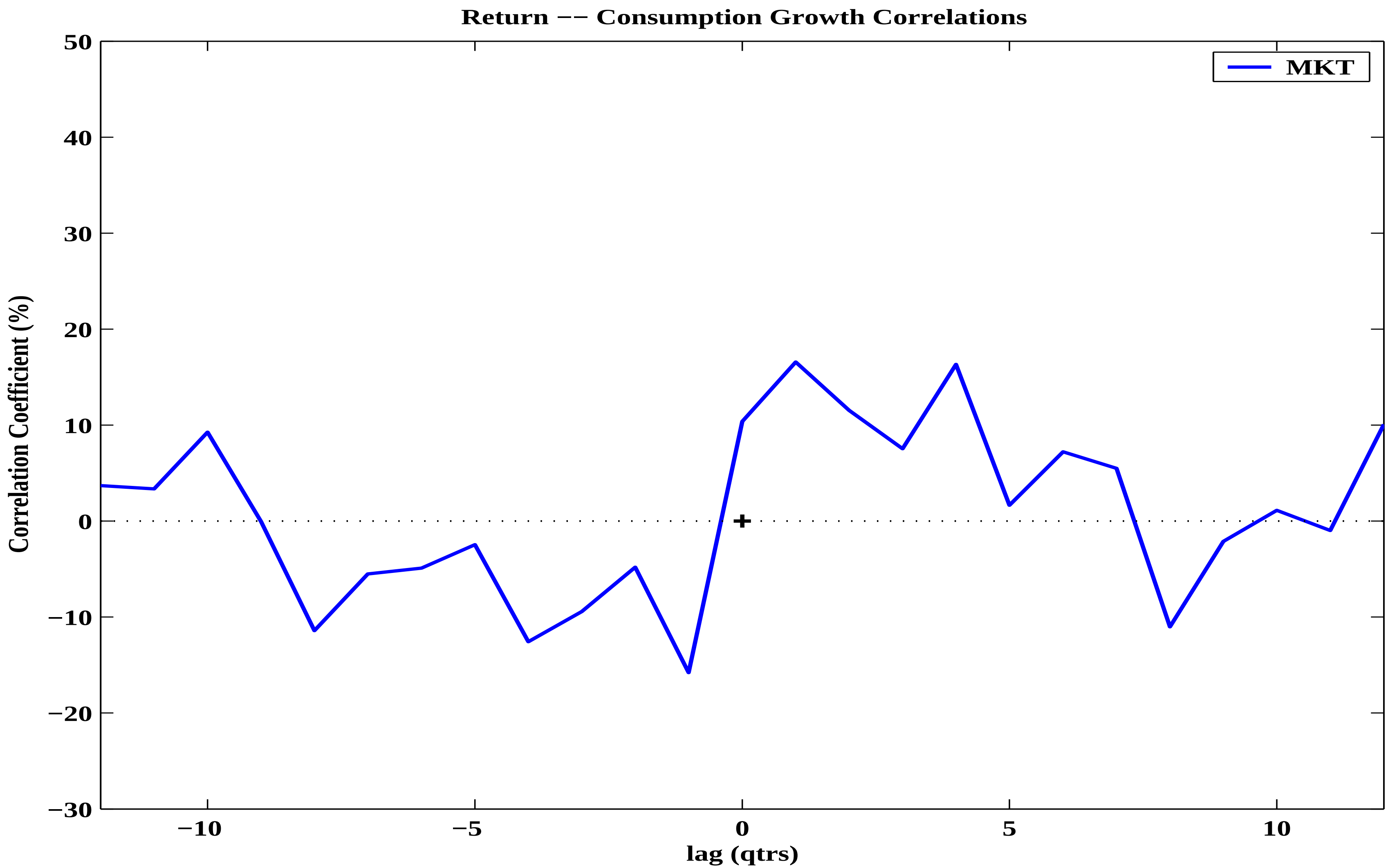
Consumption Serial Correlation



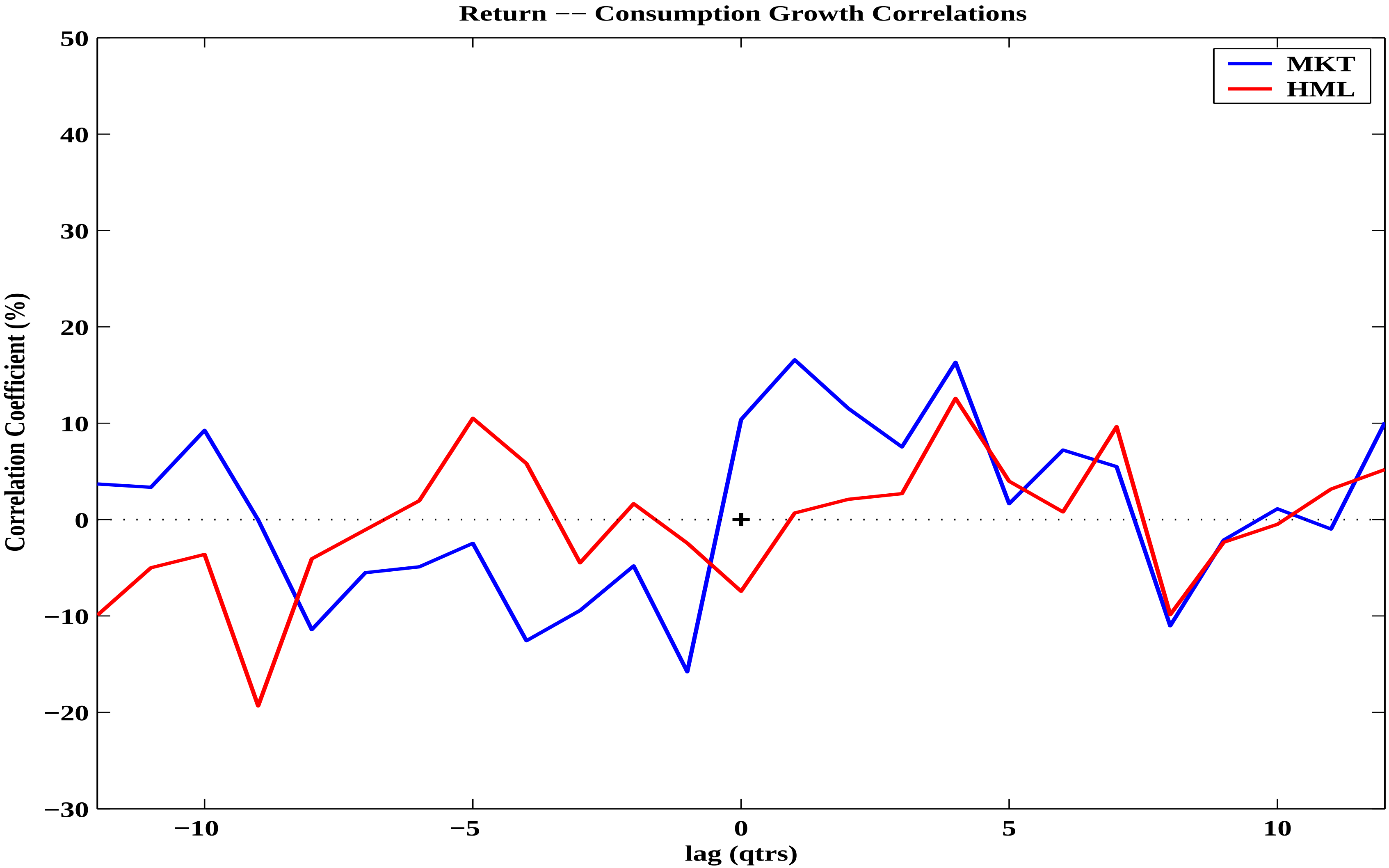
HML Return Correlation



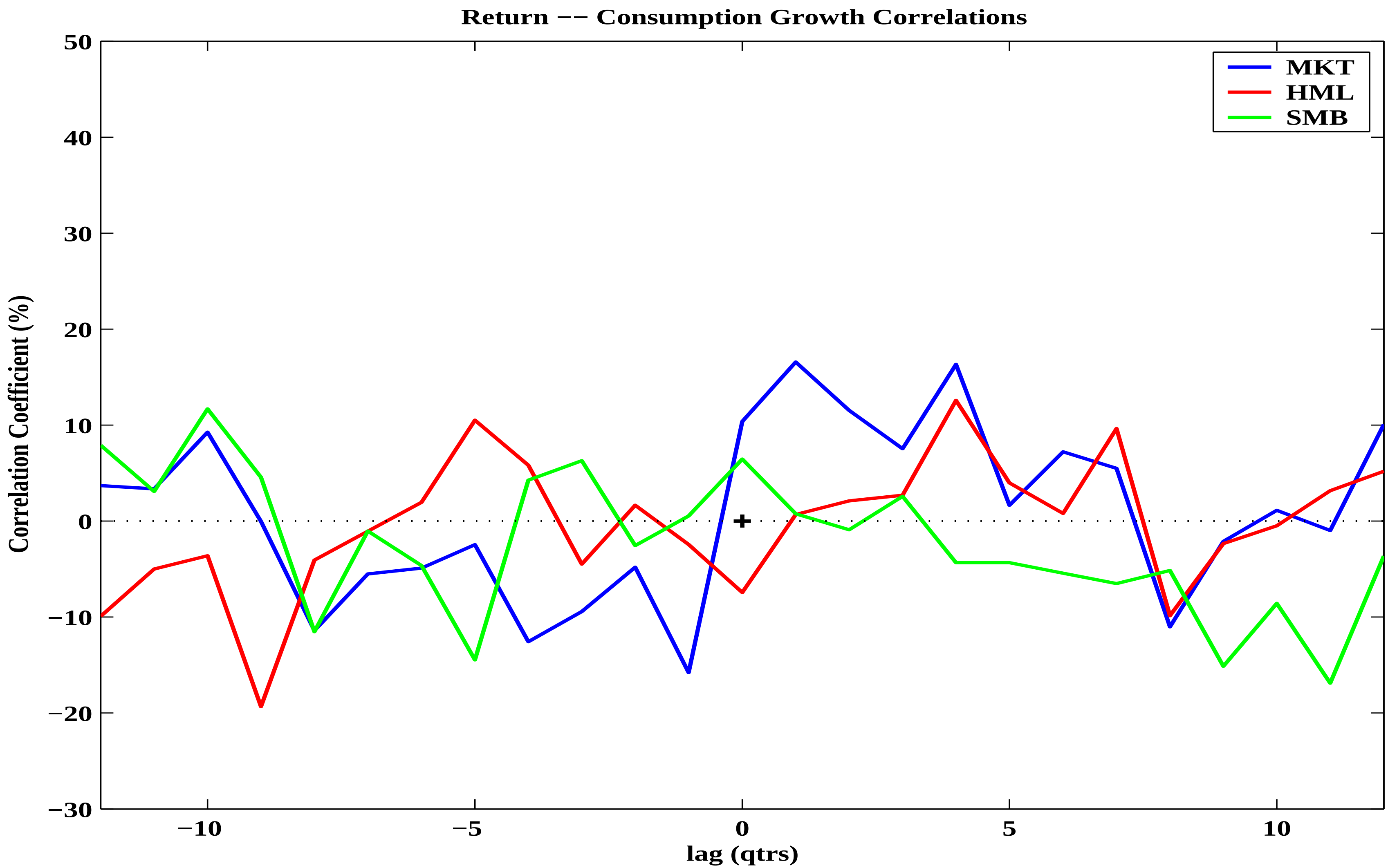
Consumption-Return Correlation



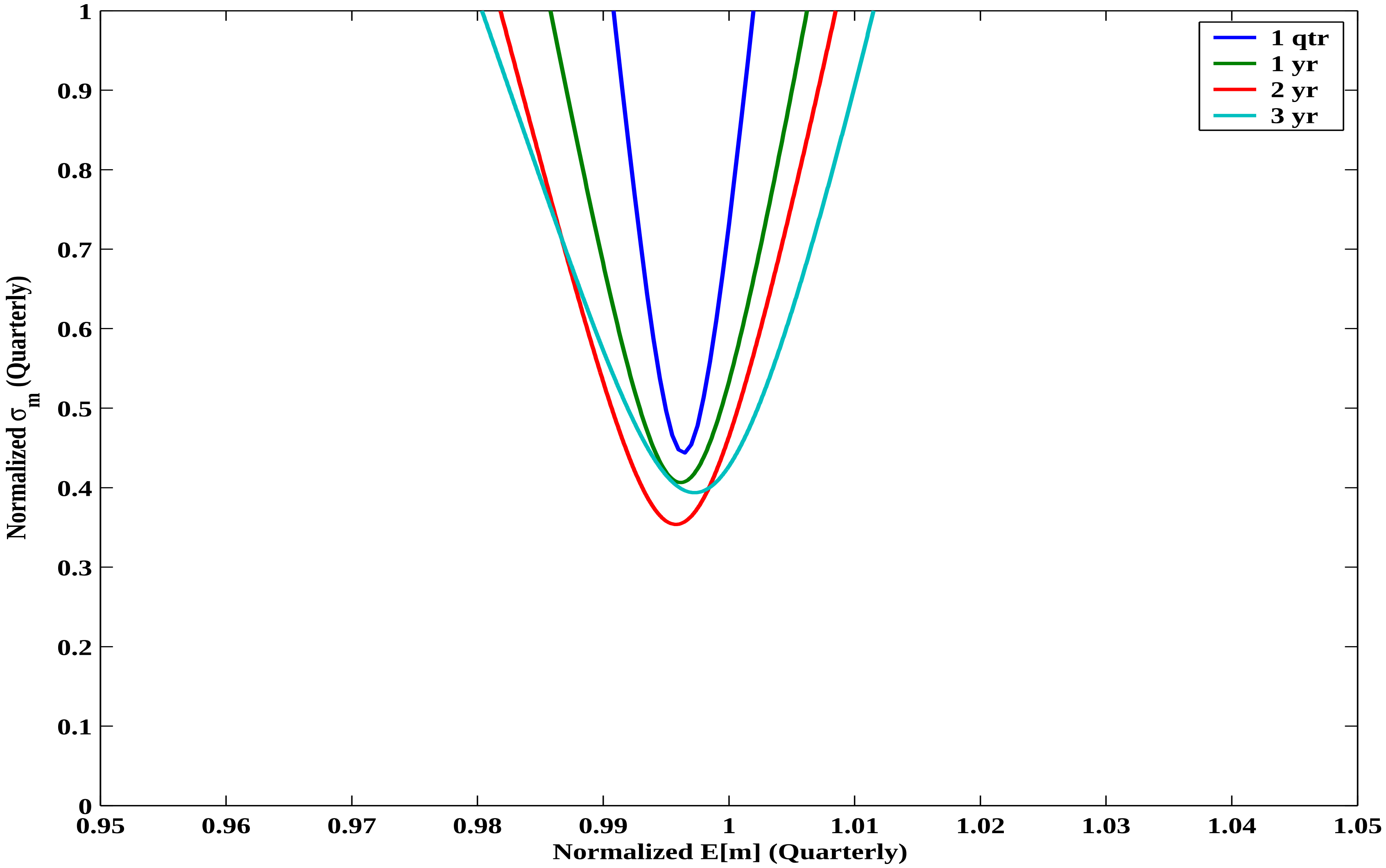
Consumption-Return Correlation



Consumption-Return Correlation



Long Horizon H-J Bounds



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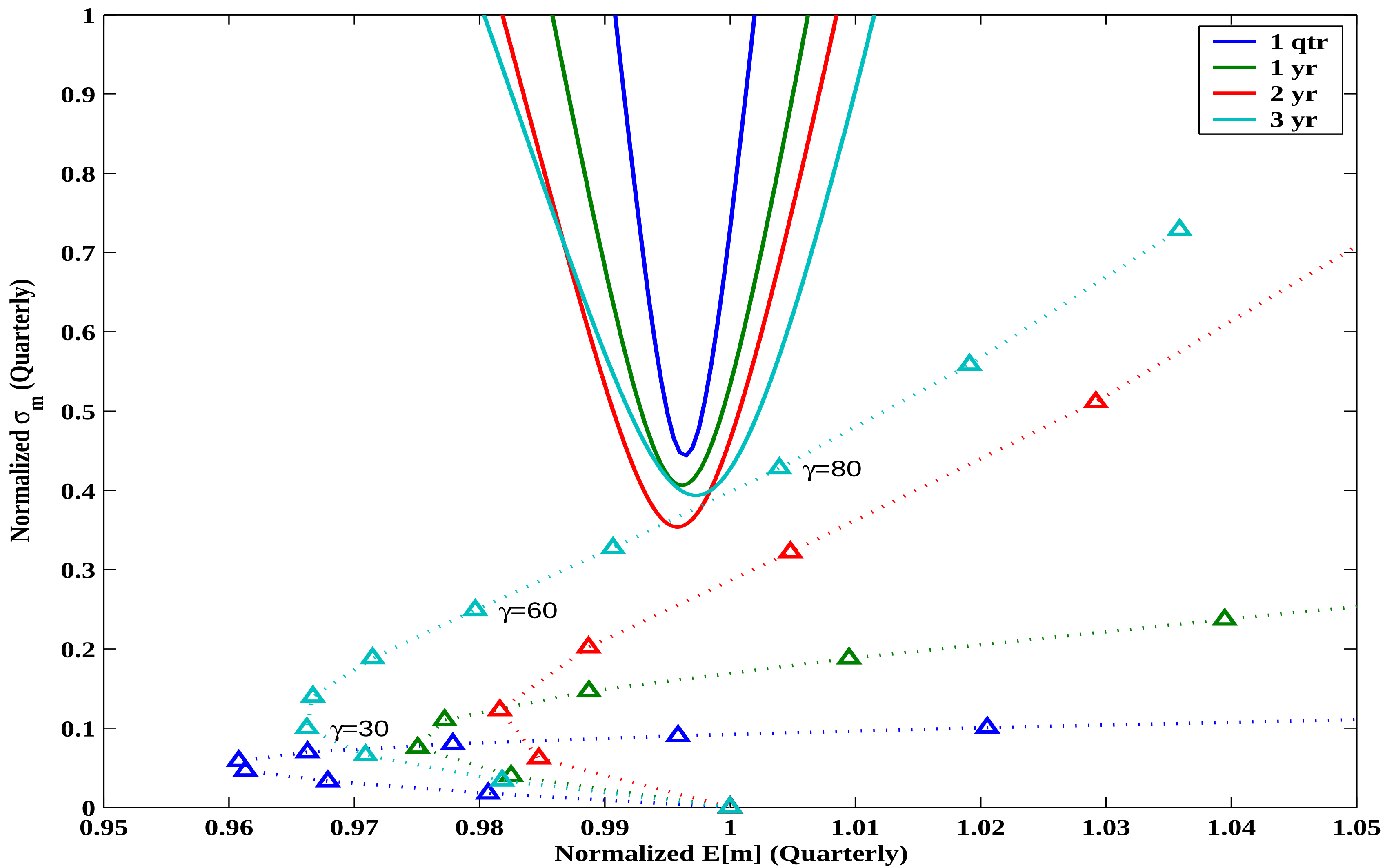

$m^* =$ projection of m onto Mkt, Rf, H and L

PG C-CAPM

- Start with the covariance expansion (valid for all τs):

$$cov(m,R) = \underbrace{E[mR]}_{=1} - E[m]E[R]$$

- Rearrange, and use $m = 1 - \gamma\Delta c$

$$\begin{aligned} E[R] &= \frac{1}{E[m]} - \frac{\sigma_m^2}{E[m]} \frac{cov(m,R)}{\sigma_m^2} \\ &= R_f + \frac{\gamma^2\sigma_c^2}{E[1-\gamma\Delta c]} \frac{\gamma cov(\Delta c, R)}{\gamma^2\sigma_c^2} \\ &= R_f + \underbrace{\frac{\gamma\sigma_c^2}{E[1-\gamma\Delta c]}}_{\equiv\lambda_S} \underbrace{\frac{cov(\Delta c, R)}{\sigma_c^2}}_{\equiv\beta_{i,S}} \end{aligned}$$

PG C-CAPM (2)

- PG estimate FM regressions to get λ_S Then, they invert the relation:

$$\lambda_s = \frac{\sigma_m^2}{E[m]} = \frac{\gamma^2 \sigma_c^2}{E[1-\gamma\Delta c]}$$

to infer the CRRA (γ) from the Fama-MacBeth slope coefficient.

- However, the log-linearization doesn't work very well in approximating $E[m]$, especially at long horizons, and for large values of γ:
  - The next term in the expansion is:

$$m = 1 - \gamma\Delta c + \frac{\gamma^2}{2}(\Delta c)^2 + \ldots$$

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