Discussion of:

Equilibrium Cross-Section of Returns

by Joao Gomes, Leonid Kogan, and Lu Zhang

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Discussant:

Kent Daniel
Kellogg School, Northwestern

The Basic Idea:

- Empirically, the CAPM has a hard time explaining size and book-to-market effects.
  - Size and book-to-market appear to capture separate risk-factors.

Gomes, Kogan and Zhang develop a model in which:

1. Firm value is attributable to assets-in-place and growth options.
   - Firm size is a proxy for the fraction of a firm's value that is attributable to assets-in-place.
     - Growth options are riskier (higher β) than assets-in-place, so in the model small firms have more growth options, and hence are riskier and earn higher returns in equilibrium.
   - Book-to-market is a proxy for the profitability of a firm's assets-in-place
     - Higher book-to-market firms have lower current profitability, but higher β, so they earn higher returns in equilibrium.
2. Conditional CAPM expains size/book-to-market premia.

The Model

Household Sector:

- Representative Household with Power Utility
  - CRRA= γ
- No labor income; no frictions.

Production Sector:

1. **Projects:**
   - Projects Arrive Randomly
     - Arrival Process is the same for all firms: large, small, growth, or value.
   - Project Cashflows are:

$$= X_{i,t} \cdot k_i = exp(x_t) \cdot \epsilon_{i,t} \cdot k_i$$

     - x_t, $\epsilon_{i,t}$ are economy-wide and firm-specific productivity processes – they are independent mean reverting processes.
     - k_i is project $scale$, which is same across firms.
   - Projects expire randomly – Poisson process with (common) arrival rate δ.
   - Cost of initiation $e_{i,t}$ differs across projects.
2. **Firms:**
   - Firm value is the sum of the ongoing projects ("assets-in-place"), and potential new projects ("growth-options")
   - Firms invest optimally – no asymmetric information or agency problems.
   - Within a firm, $\epsilon_{i,t}$ is identical across projects

Firm Value Decomposition

Firm Value is:

$$V_{ft} = \underbrace{\int_{\mathcal{I}_{ft}} \frac{k_i}{K_t} \left[\widetilde{V}_t^a (\epsilon_{it} - 1) + V_t^a \right] di}_{V_{ft}^a} + \underbrace{\frac{1}{\int_{\mathcal{F}} 1 df} V_t^o}_{V_{ft}^o}$$

There are three components:

1. $\frac{1}{K_t} V_t^a$ is the value of one extra unit of scale, for an average firm.
   - $\frac{k_i}{K_t} V_t^a$ would be the value of its assets-in-place, were it an average firm.
2. $\frac{1}{K_t} \widetilde{V}_t^a$ is the extra value per unit of scale a firm gains per unit of firm-specific productivity (ϵ_{it}).
3. V_{ft}^o is the value of the firm's growth options – this is identical across firms.

Firm Value Decomposition - Graphically

Firm Beta Decomposition

The firm's β is a weighted average of the βs of the assets-in-place and the growth options:

$$\begin{aligned}
\beta_{ft} &= \frac{V^a_{ft}}{V_{ft}}\,\beta^a_{ft} + \frac{V^o_{ft}}{V_{ft}}\,\beta^o_{ft} \\
&= \widetilde{\beta}^a_t + \frac{V^a_{ft}}{V_{ft}}\left(\beta^a_t - \widetilde{\beta}^a_t\right) + \frac{V^o_{ft}}{V_{ft}}\left(\beta^o_t - \widetilde{\beta}^a_t\right) \\
&= \widetilde{\beta}^a_t + \frac{K_{ft}}{V_{ft}}\left(\frac{K_t}{V^a_t}\right)^{-1}\left(\beta^a_t - \widetilde{\beta}^a_t\right) + \frac{V^o_{ft}}{V_{ft}}\left(\beta^o_t - \widetilde{\beta}^a_t\right)
\end{aligned}$$

where:

1. $\widetilde{\beta}^a_t$ is the average β of assets-in-place in the economy ($i.e.,$ the beta of $\widetilde{V}^a_t$).
2. β^a_t is the incremental beta due to the higher productivity ($i.e.,$ the beta of V^a_t).
3. β^o_t is the beta of growth options.

- The last term shows that small firms will have higher beta *if* $\beta^o_t - \widetilde{\beta}^a_t$.
  - Since growth options are levered, they should have higher betas.
- The second term shows that high book-to-market firms will have higher betas if $\beta^a_t - \widetilde{\beta}^a_t$
  - That is, if the betas of higher productivity firms are *lower* than of lower productivity firms.

Additional Model Implications:

Some model implications are inconsistent with empirical findings:

1. Market Sharpe Ratio relatively constant over time:
   - The simulation generates considerable variation in expected market return
   - However, there is simultaneous large variability in the market return volatity – the market sharpe ratio is relatively constant.
   - Empirically, we see dramatic variation in the market Sharpe ratio.
2. Market is MVE portfolio:
   - MacKinlay (1995) shows that combinations of Fama-French SMB, HML and Mkt portfolios generate Sharpe-ratios far in excess of market's.
3. High Variability in Consumption Growth:
   - To generate high Sharpe ratios with power utility, you need counterfactually high consumption growth variance:

$$\begin{aligned} \frac{\sigma_m}{E[m]} &= \frac{E[R_{MVE}] - r_f}{\sigma_{MVE}} \\ \gamma\sigma_{cg} &\approx SR_{MVE} \end{aligned}$$

4. Physical size (as opposed to Market Equity) forecasts future returns:
   - Berk (2000) finds that, empirically, physical size measures don't forecast future returns.

Can a Conditional CAPM Explain the Data?

- The Conditional CAPM implies that:

$$E_{t-1}[R^e_{i,t}] = \beta_{i,t-1} E_{t-1}[R^e_{vw,t}]$$

- Taking unconditional expectations, and using the definition of the covariance:

$$E[R^e_{i,t}] = E[\beta_{i,t-1}] \cdot E[R^e_{vw,t}] + cov(\beta_{i,t-1}, E_{t-1}[R^e_{vw,t}])$$

- However, if we test the CAPM unconditionally:

$$E[R^e_{i,t}] = \alpha_i^{\text{uncond}} + E[\beta_{i,t-1}] \cdot E[R^e_{vw,t}]$$

  we may reject the CAPM (find $\alpha_i^{\text{uncond}} \neq 0$) *even though the conditional CAPM holds.*

- For example, it could be that, for the Fama-French HML portfolio, $cov(\beta_{HML,t-1}, E_{t-1}[R^e_{vw,t}])$ is large.

- However, this would imply that:

$$\alpha_{HML}^{\text{uncond}} = -cov(\beta_{HML,t-1}, E_{t-1}[R^e_{vw,t}])$$

  and, using the triangle inequality:

$$\sigma_\beta \geq \frac{\alpha}{\sigma(E[R_{VW}])}$$

- The standard deviation in conditional beta that is required to satisfy this inequality is about 0.7 (/quarter), but empirically $\sigma_\beta < 0.1$.

Conclusions:

- Beautifully done model
- However, some implications seem inconsistent with other empirical findings:

  1. Power utility specification can't simultaneously explain high equity premium and premium variability, and low and steady consumption growth volatility (Campbell and Cochrane (1999)).
     - This variability is also necessary to explain cross sectional results.
  2. Model doesn't capture high Sharpe-ratios possible with value and size strategies.
  3. Book-to-market story seems tenuous.
     - Model implies that higher book-to-market firms have *higher* future dividend/profit growth.
     - Book-to-market risk effect story should be firmed up.
  4. Required level and business-cycle variability in small and value firm risk ($\beta_{i,t}$) doesn't seem to be there, empirically.

References

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